

MATH 301-01/ Exam#2/ Duration=2 Hours

1. Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2-3)}\right\}$ and $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+3)}\right\}$.

2. Evaluate $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)(s^2+9)}\right\}$.

3. Evaluate $\mathcal{L}\{t \sin(3t)\}$ and $\mathcal{L}\{t^2 \sin(3t)\}$.

4. Consider the IVP: $y''(t) + k^2y(t) = \delta_a(t)$, $y(0) = 1$ and $y'(0) = k$, where k and a are given real numbers.

Show that $Y(s) = \mathcal{L}\{y(t)\} = \frac{k + s + e^{-as}}{s^2 + k^2}$ and then solve the IVP.

5. Show that $\sin(2x)$ and $\sin(3x)$ are orthogonal functions on $[-\pi, \pi]$ and calculate the norm of $\cos(5x)$.

6. Expand $f(x) = \pi - x$ in a half-range sine series in $[0, \pi]$ and show that $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$.

7. Find the complex Fourier series of the function $f(x) = e^x$ in $[-\pi, \pi]$ and show that $\sum_{-\infty}^{\infty} \frac{(-1)^n}{n^2+1} = \frac{\pi}{\sinh(\pi)}$.

8. Find the eigenvalues and eigenfunctions of the BVP:

$$y''(x) + 5y'(x) + \lambda y(x) = 0, \quad y(0) = 0, y(4) = 0.$$

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Term 031

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