MATH 102 - Quiz 3 Section number: Student ID:

Instructions: You are required to attempt all questions. Each is worth 5 points.

1. Evaluate $\displaystyle\int \frac{2x+1}{4x^2+12x-7}dx$

Solution:
$\frac{2x+1}{4x^2+12x-7} = \frac{2x+1}{4x^2+12x+9-16} = \frac{2x+1}{(2x+3)^2-4^2} = \frac{2x+1}{(2x-1)(2x+7)}$
$\frac{2x+1}{(2x-1)(2x+7)} = \frac{A}{2x-1} + \frac{B}{2x+7}$. This leads to:

$2 = 2A + 2B$
$1 = 7A - B$

Solving the above set of simultaneous equations and we end up with: $A = 1/4$ and $B = 3/4$

So, $\displaystyle\int \frac{2x+1}{4x^2+12x-7}dx = \frac{ln(2x-1)}{8} + \frac{3ln(2x+7)}{8}$

2. Is $\displaystyle\int_1^2 \frac{1}{2x-1}dx$ an improper integral? State why or why not.

Solution:
The asymptote for the integrand is at $x = 1/2$. For interval $[1, 2]$, the integrand has no infinite discontinuity. In addition, the interval is not infinite. Hence, it is not an improper integral.

3. Evaluate $\displaystyle\int x^2cos(mx)dx$

Solution:
The goal is to try and simplify the integral. Set $u = x^2$ and $dv = cos(mx)dx$
We end up with $du = 2xdx$ and $v = \frac{1}{m}sin(mx)$. Now applying the integration by parts method, we obtain:
$\displaystyle\int x^2cos(mx)dx = \frac{x^2sin(mx)}{m} - \frac{2}{m}\underbrace{\int xsin(mx)dx}_{I}$
Now, we have to integrate I. Set $u = x$ and $dv = sin(mx)$. These lead to $du = x$ and $v = -\frac{1}{m}cos(mx)$
Applying the method we've just learned, integral I becomes:
$-\frac{xcos(mx)}{m} + \frac{1}{m}\displaystyle\int cos(mx)dx = -\frac{xcos(mx)}{m} + \frac{sin(mx)}{m^2}$
Hence, we get $\frac{x^2sin(mx)}{m} - \frac{2}{m}[-\frac{xcos(mx)}{m} + \frac{sin(mx)}{m^2}]$
$\Rightarrow \frac{x^2sin(mx)}{m} + \frac{2xcos(mx)}{m^2} - \frac{2sin(mx)}{m^3}$