Section 8.2 - Area of surface of revolution

The surface area formulae applies to solids which have been obtained by the disc method.

We take our cue from the surface area of a frustum, defined to be $\pi \times (R_1 + R_2) \times s$, where s is the slant height and R_1, R_2 correspond to the radii of the top and bottom faces.

If the surface area comprises of many small frustums each with slant height Δs, then $R_1 \approx R_2$ and we get $\Sigma\ 2\pi R\Delta s$

So if the solid was obtained by rotating a curve $y = f(x)$ from $x = a$ to $x = b$ about the x-axis then

$S = \int 2\pi f(x) ds$ where $ds = \sqrt{dx^2 + dy^2}$

leading to $S = \int_a^b 2\pi f(x)\sqrt{1 + (\frac{dy}{dx})^2} dx$

Similarly if the if the solid was obtained by rotating a curve $x = g(y)$ from $y = c$ to $y = d$, about the y-axis then

$S = \int 2\pi g(y) ds$ where $ds = \sqrt{dx^2 + dy^2}$

leading to $S = \int_c^d 2\pi f(x) \sqrt{1 + (\frac{dx}{dy})^2} dy$

Example: Prove that the surface area of a cone with the same height and radius is $\pi h^2 \sqrt{2}$.

(formula for surface area of cone $= \pi r \sqrt{h^2 + r^2}$ where $h =$ height of cone and $r =$ radius of cone)

$y = x \Rightarrow dy = dx$

$S = \int_0^h 2\pi x \sqrt{1 + 1} dx = \pi h^2 \sqrt{2}$.