## Section 7.3 - Trigonometric Substitution

In this section, we seek to solve integrands containing the following algebraic expressions using trigonometric terms:

1. $\sqrt{a^2 - x^2} \rightarrow$ use $x = a\text{sin}\theta$.
   Comes from identity: $\text{sin}^2\theta + \text{cos}^2\theta = 1$

2. $\sqrt{a^2 + x^2} \rightarrow$ use $x = a\text{tan}\theta$.
   Comes from identity: $1 + \text{tan}^2\theta = \text{sec}^2\theta$

3. $\sqrt{x^2 - a^2} \rightarrow$ use $x = a\text{sec}\theta$.
   Comes from identity: $\text{sec}^\theta - 1 = \text{tan}^2\theta$

Now let us go through some examples.

Example: Evaluate $\displaystyle\int \frac{x^2}{\sqrt{4x - x^2}} dx$

Solution:
Perform complete the square for $4x - x^2 \rightarrow$ $4-4+4x-x^2 = 4-(x-2)^2$. Use rule number 3, $a = 2$ and let $x - 2 = 2\sin\theta$. $dx = 2\cos\theta d\theta$

$$\int \frac{x^2}{\sqrt{4x - x^2}} = \int \frac{4 + 8\sin\theta + 4\sin^2\theta}{2\cos\theta} 2\cos\theta d\theta$$

$$= 6\theta - 8\cos\theta - \sin 2\theta$$

Example: Evaluate $\int \frac{dx}{(x^2 + 2x + 2)^2}$

Solution:
$$\int \frac{dx}{(x^2 + 2x + 2)^2} = \int \frac{dx}{((x + 1)^2 + 1)^2}$$

Let $x + 1 = \tan\theta$. Then, $dx = \sec^2\theta d\theta$. Use rule number 2, we get:

$$\int \frac{dx}{((x + 1)^2 + 1)^2} = \int \frac{\sec^2\theta d\theta}{\sec^4\theta} = \int \cos^2\theta d\theta$$

$$= \int \frac{(1 - cos(2\theta))}{2} d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4}$$