Section 7.1 - Integration by Parts

The formula is $\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$. More generally, we can have:

$$\int udv = uv - \int vdu$$

Note that the integrals above have been set as indefinite integrals. Putting the limits of integration are also allowed.

So the challenge is to determine which is u and v? There is no formula to solve this but always remember the golden rule: these techniques are meant to simplify, not complexify.

Example: Evaluate $\int x^2 cos(mx)dx$

Solution:
The goal is to try and simplify the integral.

Set $u = x^2$ and $dv = cos(mx)dx$

We end up with $du = 2xdx$ and $v = \frac{1}{m}sin(mx)$. Now applying the integration by parts method, we obtain:

$$\int x^2 cos(mx)dx = \frac{x^2 sin(mx)}{m} - \frac{2}{m} \underbrace{\int x sin(mx)dx}_{I}$$

Now, we have to integrate I. Set $u = x$ and $dv = sin(mx)$. These lead to $du = dx$ and $v = -\frac{1}{m}cos(mx)$

Applying the method we've just learned, integral I becomes:

$$-\frac{xcos(mx)}{m} + \frac{1}{m}\int cos(mx)dx = -\frac{xcos(mx)}{m} + \frac{sin(mx)}{m^2}$$

Hence, we get $\frac{x^2 sin(mx)}{m} - \frac{2}{m}[-\frac{xcos(mx)}{m} + \frac{sin(mx)}{m^2}]$

$\Rightarrow \frac{x^2 sin(mx)}{m} + \frac{2xcos(mx)}{m^2} - \frac{2sin(mx)}{m^3}$

Example: Using the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = e^x, \ x = 0, \ y = \pi$ about the x-axis.

Solution:
Recall the formula: $V = \int_c^d 2\pi y g(y) dy$ and sketch the region.

The limits of integration are: $c = 1$ and $d = \pi$. The function (in terms of y) is $x = g(y) = ln(y)$.

So, $V = \int_1^{\pi} yln(y)dy$

Now we can use what we have learned here and set $u = ln(y)$ and $dv = ydy$. We end up with: $du = \frac{dy}{y}$ and $v = \frac{y^2}{2}$ (note this is small v).

$$V = [\frac{y^2 ln(y)}{2}]_1^{\pi)} - \int_1^{\pi} \frac{ydy}{2}$$

$$V = [\frac{y^2 ln(y)}{2}]_1^{\pi} - [\frac{y^2}{4}]_1^{\pi}$$

$$V = \frac{\pi^2 ln(\pi)}{2} - \frac{\pi^2}{4} + \frac{1}{4} = 111.97 \text{ cubic units}$$