

Section 6.5 - Average Value of a function

The average value of a function $f(x)$ on the interval $[a, b]$ is defined by

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

Example: (a) Find the average value of $f(x) = \sqrt{x}$ on $[0, 4]$.

(b) Find c such that $f(c) = f_{ave}$.

(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f .

Solution:

$$(a) f_{ave} = \frac{1}{4} \int_0^4 \sqrt{x} dx = \frac{2(4)^{3/2}}{3} = \frac{8}{3} = \frac{4}{3}$$

$$(b) \sqrt{c} = 4/3 \rightarrow c = 16/9$$

(c) Using the mean-value theorem for integrals, the rectangle has length of 4 and width of $256/9$.

Example: Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

$$f_{ave} = 3 = \frac{1}{b} \int_0^b (2 + 6x - 3x^2) dx$$

$$3 = \frac{1}{b} [2b + 3b^2 - b^3]$$

$$3 = 2 + 3b - b^2 \rightarrow b^2 - 3b + 1 = 0 \rightarrow b = \frac{3 \pm \sqrt{5}}{2}$$