Section 6.3 - Volume by cylindrical shells

Now, we go through the final method: shell method.

It seems very complicated and difficult to understand but hang in there!!

As the name suggests, we are talking about a shell. Imagine a very thin piece of metal (but not so thin like paper), like a thin rectangular cube.

It has length, width and height.

Now imagine curling this piece of metal so that it looks like a cylindrical shell (like a cup).

We can construct any volume comprising of a series of these cylindrical shells and that's the essence of this method. Now let's express this mathematically. The textbook has lots of amazing pictures so use them as you go through these notes.

If you un-curl this shell, you will see that the length of this rectangular cube is the circumference. How did we get this circumference you say? We use a sample point, x^* and the circumference $= 2\pi x^*$.

The width of this rectanglar cube is Δx. How do I know this? Imagine that you are somehow able to flatten this piece of metal. The "extra stuff" you see as you're doing it is the Δx. This is fixed.

The height of this rectangular cube is simply $f(x^*)$. It is this variation in height which is responsible for the series of cylindrical shells formed to comprise the overall volume.

Hence, $V = \int_a^b 2\pi x f(x) dx$ should the axis of rotation be the **y-axis**. Otherwise, $V =$

$\int_c^d 2\pi y g(y) dy$ if the **x-axis** is the axis of rotation.

Example: Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $x = 0$ and $y = 8$ about the x-axis.

Solution:
Sketch the region of interest first.

Shell method:
$V = \int_0^8 2\pi y(y^{1/3}) dy = \int_0^8 2\pi y^{4/3} dy$
$V = 2\pi \frac{3}{7}(8)^{7/3} = \frac{768\pi}{7}$

Washer method:
This method is appropriate because the region is bounded between 2 functions where

one of them isn't the x-axis. Otherwise, we'd use the disc method.

$$V = \int_0^2 \pi[8^2 - (x^3)^2]dx = 128\pi - \frac{128\pi}{7} = \frac{768\pi}{7}$$

So, we see that the two appropriate methods here lead to the same conclusion.