Section 11.5 - Alternating Series

Alternating Series Test

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots \text{ satisfies}$$

(i) $b_{n+1} \leq b_n$ for all n. This means that b_n's are decreasing.

(ii) $\lim_{n\to\infty} b_n = 0$

then the series is convergent.

Example: Test the alternating series
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2\sqrt{n}}$$
for convergence or divergence.

$$\lim_{n\to\infty} \frac{\sqrt{n}}{1+2\sqrt{n}} = \frac{1}{2} \neq 0$$

So it is a DIVERGENT SERIES.

Example: Test the alternating series $\sum_{n=1}^{\infty}(-1)^n \frac{ln(n)}{n}$ for convergence or divergence.

$$\lim_{n \to \infty} \frac{ln(n)}{n} = 0$$

$b_{n+1} = \frac{ln(n+1)}{n+1}$ and so we are left with showing that $b_{n+1} \leq b_n$ for all n.

If $b_n = \frac{ln(n)}{n}$, then $b_n' = \frac{1-ln(n)}{n^2} \leq 0$ for $n \geq 3$ $\Rightarrow$ b_n is decreasing. You might be thinking how about for $n = 1$ and $n = 2$? $b_n' > 0$ at these two values. However, the series still diverges since $b_n' > 0$ for only two terms of the series. If we split the series into $\sum_{n=1}^{\infty}(-1)^n \frac{ln(n)}{n} = \sum_{n=1}^{2}(-1)^n \frac{ln(n)}{n} + \sum_{n=3}^{\infty}(-1)^n \frac{ln(n)}{n}$, then we see that the first series converges (because only two terms, so sum is finite) while the second series diverges. So if the second series diverges, then the original series must also diverge by the Comparison theorem.

Thus, the alternating series CONVERGES!!