Section 11.4 - The Comparison Tests

The Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

(i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

(ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

Limit Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both series diverge.

Example: Determine whether $\displaystyle\sum_{n=0}^{\infty} \frac{1 + sin(n)}{10^n}$ converges or diverges.

Let $a_n = \frac{1+sin(n)}{10^n}$ and $b_n = \frac{2}{10^n}$. We make this choice because we know $\displaystyle\sum_{n=0}^{\infty} b_n$ converges by the geometric series test since $r = \frac{1}{10} < 1$.

Since $a_n < b_n$ then by Comparison test, $\displaystyle\sum_{n=0}^{\infty} a_n$ converges.

Example: Determine whether $\displaystyle\sum_{n=0}^{\infty} (1+\frac{1}{n})^2 e^{-n}$ converges or diverges.

Let $a_n = (1 + \frac{1}{n})^2 e^{-n}$ and $b_n = 2^2 e^{-n}$. We make this choice because we know $\displaystyle\sum_{n=0}^{\infty} b_n$ converges by the geometric series test since

$r = \frac{1}{e} < 1$. Let us try to use the limit comparison test.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = (1 + 1/n)^2 = 1 > 0.$$

Since $a_n < b_n$ then by the limit comparison test, $\sum_{n=0}^{\infty} a_n$ converges.