Section 11.1 - Sequences

A sequence is a list of numbers written in a definite order. They are usually given by a formula.

Theorem: If $\lim_{x \to \infty} f(x) = L$ and $f(n) = a_n$ where n is an integer, then $\lim_{n \to \infty} a_n = L$.

Definition: A sequence a_n is decreasing and increasing if $a_n > a_{n+1}$ and $a_n < a_{n+1}$ for $n \geq 1$ respectively. It is monotonic if the sequence is either decreasing or increasing. An example of a non-monotonic function is an alternating sequence (i.e. sign changes with consecutive n).

Definition: A bounded sequence means that there is a limit to how the values of the sequence can go (either below or above).

Theorem: Every bounded, monotonic sequence is convergent.

Example: Does the sequence a_n such that $a_n = \frac{\sqrt{n}}{1+\sqrt{n}}$ converge or diverge. If it converges, then find the limit.

Solution:
$a_n = \frac{1}{1/\sqrt{n}+1}$. Then, $\lim_{n\to\infty} a_n = \dfrac{1}{0+1} = 1$.

Example: Does the sequence a_n such that $a_n = \frac{e^n+e^{-n}}{e^{2n}-1}$ converge or diverge. If it converges, then find the limit.

Solution:
$a_n = \frac{1+e^{-2n}}{e^n-e^{-n}}$. Then, $\lim_{n\to\infty} a_n = \dfrac{1+0}{\infty-0} = 0$.

Example: Determine whether the sequence a_n where $a_n = n + \frac{1}{n}$ is increasing, decreasing or non-monotonic. Is the sequence bounded?

Solution:
The sequence a_n is increasing but it is not bounded.