1. [**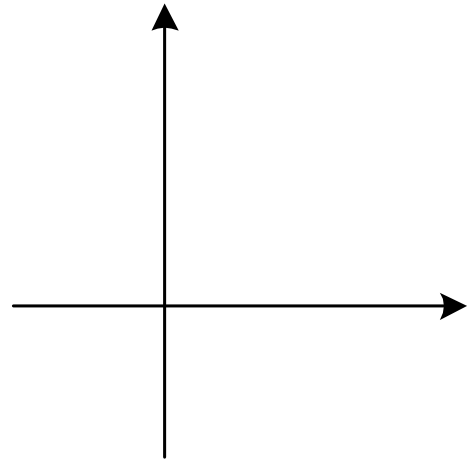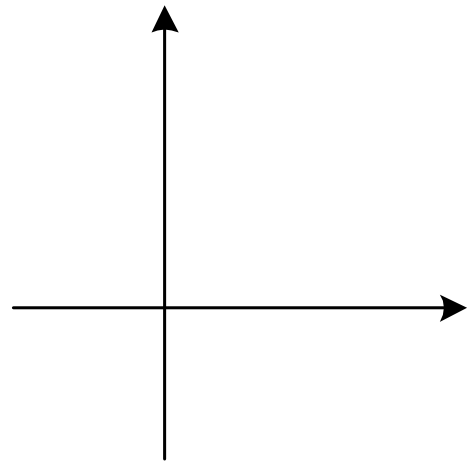6 + 6 points**] Using the method of Cylindrical Shells, **set up**, but **do not evaluate**, an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = e^{2x}$, $y = e^2$, and $x = 0$

   (a) about the line $x = -2$ [Sketch the region and a typical rectangle]

   (b) about the x-axis.   [Sketch the region and a typical rectangle]

2. [**7 points**] Find the average value of the function $f(x) = x \sec^2(2x)$ on the interval $\left[0, \dfrac{\pi}{8}\right]$.

3. Evaluate each of the following integrals:

   (a) **[6 points]** $\int \sin^5(3t) \cos^4(3t) \, dt$.

   (b) **[6 points]** $\int \dfrac{\sqrt{x}}{x+4} \, dx$.

   (c) **[8 points]** $\int \dfrac{1}{1+2\sin x} \, dx$.   Hint: Use the substitution $t = \tan\left(\dfrac{x}{2}\right)$.

(d) [**7 points**] $\displaystyle\int_{2/5}^{4/5} \frac{\sqrt{25x^2 - 4}}{x}\, dx.$

(e) [**9 points**] $\displaystyle\int \frac{4x}{(x+1)(x-1)^2}\, dx.$

4.   (a) Determine if the integral is improper or not. Justify.

   (i)   [**3 points**] $\int_{2}^{5} \ln(x-1)\, dx.$

   (ii) [**3 points**] $\int_{1}^{5} \dfrac{1}{x^2 - x}\, dx.$

   (b) [**6 points**] Determine if the integral $\int_{-\infty}^{0} \dfrac{1}{\sqrt{3-x}}\, dx$ is convergent or divergent. If it is convergent, find its value.

5. [**6 points**] Determine whether the sequence $\left\{ \dfrac{\sin(3n)}{3n+1} \right\}_{n=1}^{+\infty}$ converges or diverges. If it converges, find its limit.

6. Consider the series $\displaystyle\sum_{n=1}^{+\infty} \dfrac{\ln(n+2) - \ln(n+1)}{\ln(n+1) \cdot \ln(n+2)}$.

   (a) [**5 points**] Find a formula for S_n, the sequence of partial sums.

   (b) [**4 points**] Determine whether the given series converges or diverges. If it converges, find its sum.

7. Determine whether the series converges or diverges. Justify your answer.

(a) [**5 points**] $\displaystyle\sum_{n=1}^{+\infty} \sin^{-1}\left(\frac{n}{n+1}\right)$.

(b) [**5 points**] $\displaystyle\sum_{n=0}^{+\infty} \frac{(2e)^{n+1}}{3^{2n-1}}$.

(c) [**8 points**] $\displaystyle\sum_{n=1}^{+\infty} n\, e^{-2n}$.