

HW 1. Q 2.

Branch cut structure for $w = \ln [4 + \sqrt{z^2 - 9}]$.

Soln.

Write $w = \ln [4 + \zeta]$ where $\zeta = \sqrt{z^2 - 9}$.

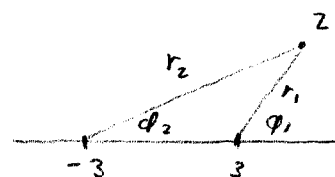
Then $w = w(\zeta)$ has the branch point $\zeta = -4$ which

corresponds to $z^2 = 25$.

For any z , we can write $z = 3 + r_1 e^{i\phi} = -3 + r_2 e^{i\phi_2}$ as shown in the figure. Accordingly,

the function $\zeta = \zeta(z)$ has the two branches

$$\zeta_{\pm}(z) = \pm \sqrt{r_1 r_2} e^{i(\phi_1 + \phi_2)/2},$$



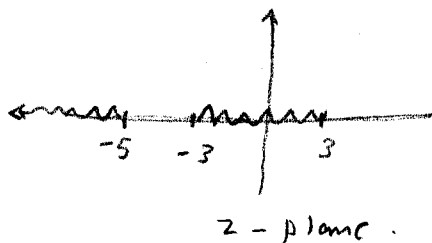
with

$\zeta_{\pm}(z)$ maps the + real $z > 3$ into the \pm real axis.

$\sim \sim \sim - \sim z < -3 \sim \sim + \sim \sim$.

In particular, $\zeta_{+}(-5) = \zeta_{-}(5) = -4$.

Therefore for + branch of ζ a branch cut of w is



For the - branch of ζ a branch cut of w is

