HW 1.2

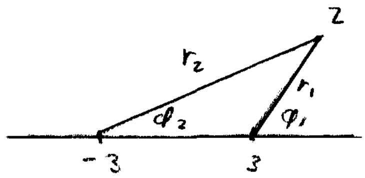
Branch cut structure for $w = \ln\left[4 + \sqrt{z^2-9}\right]$.

Soln.

Write $w = \ln[4+\zeta]$ where $\zeta = \sqrt{z^2-9}$.

Then $w = w(\zeta)$ has the branch point $\zeta = -4$ which corresponds to $z^2 = 25$.

For any z, we can write $z = 3 + r_1 e^{i\varphi_1} = -3 + r_2 e^{i\phi_2}$ as shown in the figure. Accordingly, the function $\zeta = \zeta(z)$ has the two branches



$$\zeta_\pm(z) = \pm\sqrt{r_1 r_2}\, e^{i(\varphi_1+\varphi_2)/2},$$

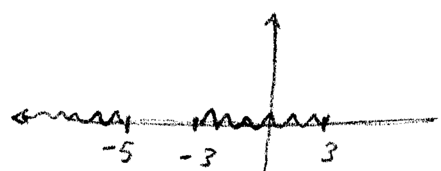
with

$\zeta_\pm(z)$ maps the + real $z > 3$ into the $\pm$ real axis.

~ ~ ~ − ~ $z < -3$ ~ ~ $\mp$ ~ ~ .

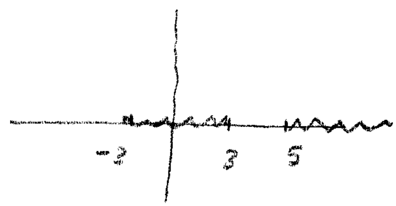
In particular, $\zeta_+(-5) = \zeta_-(5) = -4$.

Therefore for + branch of ζ a branch cut of w is



z - plane.

For the − branch of ζ a branch cut of w is



z - plane.