

**Math 514 (092)**  
**Final**  
**Due: Sunday Jun 13**

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1. Use Watson's lemma to obtain an asymptotic expansion of

$$f(x) = \int_{-\infty}^{\infty} e^{-x \cosh t} dt.$$

2. Use integration by parts to find an asymptotic expansion for large  $x$  of

$$f(x) = \int_1^{\infty} \frac{e^{-xt}}{t^2} dt$$

and prove that the expansion is asymptotic.

3. Use the inversion formula to find  $\mathcal{L}^{-1}\left\{\frac{\cos s}{s}\right\}$ .

4. Use Fourier sine transform to solve

$$\begin{aligned} u_{xx} + u_{yy} &= 0, & -\infty < x < \infty, \quad y > 0, \\ u(x, 0) &= e^{-x}, & -\infty < x < \infty. \end{aligned}$$

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