

Math 513 (071) / Exam 2

Show your Work.

Total Grade: 25

Time: 120 min

1. Show that $\{\sin nx\}_{n=1}^{\infty}$ is an orthogonal set on $[0, \pi]$. Find the norm of each function.

2. Find the first three non-vanishing coefficients in the Legendre polynomial expansion for

$$f(x) = \begin{cases} x, & -1 < x < 0, \\ 0, & 0 < x < 1. \end{cases}$$

3. Solve: Use d'Alembert formula to solve:

$$u_{tt} = u_{xx}, \quad -\infty < x < \infty, \quad 0 < t$$

$$u(x,0) = \begin{cases} 1 - |x|, & |x| \leq 1, \\ 0, & |x| > 1. \end{cases}$$

$$u_t(x,0) = 0, \quad -\infty < x < \infty$$

Sketch $u(x,3)$.

4. Solve: Use separation of variables method to solve

$$u_{tt} = u_{rr} + \frac{1}{r} u_r, \quad 0 \leq r < 3, \quad t > 0$$

$$u(r,0) = 5$$

$$u_t(r,0) = 0$$

$$u(3,t) = 0$$

5. Solve: Use separation of variables method to solve

$$u_t = u_{xx}, \quad 0 \leq x < 4, \quad t > 0$$

$$u(x,0) = 0,$$

$$u(0,t) = 10$$

$$u(4,t) = 0$$