KF

Math 513 Formulas

Laplace Transform

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt$$

$\mathcal{L}\{1\}=s^{-1}$, $\mathcal{L}\{t^n\}=n!/s^{n+1}$

$\mathcal{L}\{e^{at}\}=(s-a)^{-1}$

$\mathcal{L}\{\sin kt\}=k(s^2+k^2)^{-1}$

$\mathcal{L}\{\cos kt\}=s(s^2+k^2)^{-1}$

$\mathcal{L}\{\sinh kt\}=k(s^2-k^2)^{-1}$

$\mathcal{L}\{\cosh kt\}=s(s^2-k^2)^{-1}$

$\mathcal{L}\{f(t)\}=F(s)$

$\mathcal{L}\{e^{at}f(t)\}=F(s-a)$

$\mathcal{L}\{f(t-a)H(t-a)\}=e^{-as}F(s)$

$\mathcal{L}\{f(t)H(t-a)\}=e^{-as}\mathcal{L}\{f(t+a)\}$

$\mathcal{L}\{t^n f(t)\}=(-1)^n F^{(n)}(s)$

$$\mathcal{L}\{f^{(n)}(t)\}=s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\cdots-f^{(n-1)}(0)$$

$$f*g=\int_0^t f(x)g(t-x)dx$$

$\mathcal{L}\{f*g\}=\mathcal{L}\{f\}\mathcal{L}\{g\}$

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\}=F(s)/s$$

$$\mathcal{L}\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt, \text{ Period } T.$$

$\mathcal{L}\{\delta(t-t_0)\}=e^{-st_0}$

$$\mathcal{L}\left\{\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)\right\}=\frac{e^{-a\sqrt{s}}}{s}$$

Fourier series expansion

$$\frac{a_0}{2}+\sum_{n=1}^\infty\left[a_n\cos\frac{n\pi t}{L}+b_n\sin\frac{n\pi t}{L}\right]$$

$$a_0=\frac{1}{L}\int_r^{r+2L}f(t)dt,\quad a_n=\frac{1}{L}\int_r^{r+2L}f(t)\cos\frac{n\pi t}{L}dt$$

$$\sum_{n=-\infty}^\infty c_ne^{i\omega t},\quad c_n=\frac{1}{2L}\int_r^{r+2L}f(t)e^{-i\omega_n t}dt,\quad \omega_n=\frac{n\pi}{L}.$$

Laplacian

$$\Delta u=u_{rr}+r^{-1}u_r+r^{-2}u_{\theta\theta}+u_{zz}$$

$$\Delta u=u_{rr}+2r^{-1}u_r+(r\sin\theta)^{-2}u_{\phi\phi}+r^{-2}u_{\theta\theta}+r^{-2}\cot\theta\, u_\theta$$

Integral Formulas

$$\int te^{at}dt=\frac{e^{at}}{a^2}(at-1)$$

$$\int t^2e^{at}dt=\frac{e^{at}}{a^2}(a^2t^2-2at+2)$$

$$\int t\sin at\,dt=\frac{\sin at-at\cos at}{a^2}$$

$$\int t\cos at\,dt=\frac{\cos at+at\sin at}{a^2}$$

Fourier Transform

$$\mathcal{F}\{f(t)\}=\int_{-\infty}^\infty f(t)e^{-i\omega t}dt=F(\omega)$$

$$\mathcal{F}^{-1}\{F(\omega)\}=\frac{1}{2\pi}\int_{-\infty}^\infty F(\omega)e^{i\omega t}d\omega=f(t)$$

$\mathcal{F}\{e^{i\omega_0 t}\}=2\pi\delta(\omega-\omega_0)$

$\mathcal{F}\{\delta(t)\}=1$

$$\mathcal{F}\{\operatorname{sgn}(t)\}=\begin{cases}2/(i\omega), & \omega\neq 0\\ 0, & \omega=0\end{cases}$$

$$\mathcal{F}\{H(t)\}=\pi\delta(\omega)+\frac{1}{i\omega}$$

$\mathcal{F}\{f(t-\tau)\}=e^{-i\omega\tau}F(\omega)$

$\mathcal{F}\{f(t)e^{i\omega_0 t}\}=F(\omega-\omega_0)$

$\mathcal{F}\{f^n(x)\}=(i\omega)^nF(\omega)$

$$f(t)*g(t)=\int_{-\infty}^\infty f(x)g(t-x)dx$$

Parseval's Inequality

$$\int_{-\infty}^\infty |f(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^\infty |F(\omega)|^2d\omega$$

Fourier-Bessel coefficients:

$$A_k=\frac{1}{C_k}\int_0^L xf(x)J_n(\mu_kx)dx.$$

If $J_n(\mu_kL)=0$, $C_k=\frac{1}{2}L^2J_{n+1}^2(\mu_kL)$

If $\mu_kJ_n'(\mu_kL)+hJ_n(\mu_kL)_i=0$,

$$C_k=\frac{\mu_k^2L^2-n^2+h^2L^2}{2\mu_k^2}J_n^2(\mu_kL)$$

Fourier-Legendre coefficients

$$c_n=\frac{2n+1}{2}\int_{-1}^1 f(x)P_n(x)dx$$

Bessel and Legendre Equations

$$x^2y''+xy'+(\mu^2x^2-n^2)y=0$$

$$(1-x^2)y''-2xy'+n(n+1)y=0$$

$$\left(x^nJ_n(x)\right)'=x^nJ_{n-1}(x)$$

$$\left(x^{-n}J_n(x)\right)'=-x^nJ_{n+1}(x)$$

$$P_0(x)=1,\ P_1(x)=x,\ P_2(x)=\frac{1}{2}(3x^2-1),$$

$$P_3(x)=\frac{1}{2}(5x^3-3x)$$

$$P_4(x)=\frac{1}{8}(35x^4-30x^2+3)$$

D'Alembert's Formula

$$u(x,t)=\frac{f(x+ct)+f(x-ct)}{2}+\frac{1}{2c}\int_{x-ct}^{x+ct}g(\tau)d\tau$$