

Math 301 Formulas

Vector Calculus

$$\text{Green's Theorem: } \oint_C P dx + Q dy = \iint_R (Q_x - P_y) dA$$

$$\text{Stokes' Theorem: } \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl} \mathbf{F} \cdot \mathbf{n} dS$$

$$\text{Divergence Theorem: } \iiint_D \mathbf{F} \cdot \mathbf{n} dS = \iiint_D \text{div} \mathbf{F} dV$$

Laplace Transform

$$\mathcal{L}\{1\} = s^{-1}$$

$$\mathcal{L}\{t^n\} = n! / s^{n+1}$$

$$\mathcal{L}\{e^{at}\} = (s - a)^{-1}$$

$$\mathcal{L}\{\sin kt\} = k(s^2 + k^2)^{-1}$$

$$\mathcal{L}\{\cos kt\} = s(s^2 + k^2)^{-1}$$

$$\mathcal{L}\{\sinh kt\} = k(s^2 - k^2)^{-1}$$

$$\mathcal{L}\{\cosh kt\} = s(s^2 - k^2)^{-1}$$

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s - a)$$

$$\mathcal{L}\{f(t - a)U(t - a)\} = e^{-as} F(s)$$

$$\mathcal{L}\{f(t)U(t - a)\} = e^{-as} \mathcal{L}\{f(t + a)\}$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

$$f * g = \int_0^t f(\tau)g(t - \tau)d\tau$$

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$$

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = F(s) / s$$

$$\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t)dt, \text{ Period } T.$$

$$\mathcal{L}\{\delta(t - t_0)\} = e^{-st_0}$$

Laplacian

$$\Delta u = u_{rr} + r^{-1}u_r + r^{-2}u_{\theta\theta} + u_{zz}$$

$$\Delta u = u_{rr} + 2r^{-1}u_r + (r \sin \theta)^{-2}u_{\phi\phi} + r^{-2}u_{\theta\theta} + r^{-2} \cot \theta u_\theta$$

Orthogonal expansions coefficients

Fourier-Bessel coefficients:

$$\text{For } J_n(\lambda_i b) = 0,$$

$$c_i = \frac{2}{b^2 J_{n+1}^2(\lambda_i b)} \int_0^b x J_n(\lambda_i x) f(x) dx$$

$$\text{For } hJ_n(\lambda_i b) + b\lambda_i J_n'(\lambda_i b) = 0,$$

$$c_i = \frac{2\lambda_i^2}{(\lambda_i^2 b^2 - n^2 + h^2) J_n^2(\lambda_i b)} \int_0^b x J_n(\lambda_i x) f(x) dx$$

Fourier coefficients:

$$\int_{-p}^p \cos^2 \frac{n\pi}{p} x dx = \int_{-p}^p \sin^2 \frac{n\pi}{p} x dx = p$$

$$c_n = \frac{1}{2p} \int_{-p}^p f(x) e^{-in\pi x/p} dx$$

Fourier-Legendre coefficients

$$c_n = \frac{2n+1}{2} \int_{-1}^1 f(x) P_n(x) dx$$

$$= \frac{2n+1}{2} \int_0^\pi f(\theta) P_n(\cos \theta) \sin \theta d\theta$$

Bessel and Legendre Equations

$$x^2 y'' + xy' + (x^2 - \nu^2)y = 0$$

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$

$$(x^n J_n(x))' = x^n J_{n-1}(x)$$

$$(x^{-n} J_n(x))' = -x^{-n} J_{n+1}(x)$$

$$P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1),$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x),$$

Fourier Transform

$$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx = F(\alpha)$$

$$\mathcal{F}^{-1}\{F(\alpha)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\alpha) e^{-i\alpha x} d\alpha = f(x)$$

$$\mathcal{F}_s\{f(x)\} = \int_0^{\infty} f(x) \sin \alpha x dx = F(\alpha)$$

$$\mathcal{F}_s^{-1}\{F(\alpha)\} = \frac{2}{\pi} \int_0^{\infty} F(\alpha) \sin \alpha x d\alpha = f(x)$$

$$\mathcal{F}\{f''(x)\} = -\alpha^2 F(\alpha)$$

$$\mathcal{F}_s\{f''(x)\} = -\alpha^2 F(\alpha) + \alpha f(0)$$

$$\mathcal{F}_c\{f''(x)\} = -\alpha^2 F(\alpha) - f'(0)$$