Math 301 (072) / Exam I (Ch 9, 4.1, 4.2)

Show your Work. **Total Grade: 25** **Time: 90 min**

1) Let $\mathbf{F}(t) = \langle x, 2x - y, z \rangle$, $C : \mathbf{r}(t) = \langle t^2, t, e^{2t} \rangle$, $0 < t < 2$. Find the work done by **F** along *C*.

2) Let $\mathbf{r}(t) = \langle t, e^t \rangle$.
   a. Graph the curve traced by $\mathbf{r}(t)$, $-3 < t < 4$.
   b. Find and graph $\mathbf{r}'(0)$.

3) Let $\mathbf{F}(x, y) = \langle ye^x + x, \sin y + e^x \rangle$
   a. Show that **F** is a gradient field.
   b. Find a potential function for **F**.
   c. Evaluate $\int_{(0,\pi)}^{(2,0)} \mathbf{F}.d\mathbf{r}$.

4) Find the Laplace transform for $f(t) = \begin{cases} 2 & 0 \le t \le 4 \\ 1 & t > 4 \end{cases}$.

5) Find $\mathcal{L}^{-1}\left\{\dfrac{3s-1}{s^2 - s}\right\}$.

6) Let S: $z = 4 - x^2 - y^2$, $z \ge 0$. Verify Stokes' theorem if $\mathbf{F}(t) = \langle xy, yz, x \rangle$

Distribution of points: Q1= 4pts, Q2= 5pts, Q3 = 6pts, Q4 = 2pts, Q5 = 3pts, Q6 = 5pts.

$\mathcal{L}\{1\} = s^{-1}$
$\mathcal{L}\{t^n\} = n!/s^{n+1}$
$\mathcal{L}\{e^{at}\} = (s-a)^{-1}$
$\mathcal{L}\{\sin kt\} = k(s^2 + k^2)^{-1}$
$\mathcal{L}\{\cos kt\} = s(s^2 + k^2)^{-1}$
$\mathcal{L}\{\sinh kt\} = k(s^2 - k^2)^{-1}$
$\mathcal{L}\{\cosh kt\} = s(s^2 - k^2)^{-1}$

Stokes' Theorem: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl}\,\mathbf{F} \cdot \mathbf{n}\, dS$

Divergence Theorem: $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_D \text{div}\,\mathbf{F}\, dV$