

Math 301 Formulas

Laplace Transform

$$\mathcal{L}\{1\} = s^{-1}$$

$$\mathcal{L}\{t^n\} = n! / s^{n+1}$$

$$\mathcal{L}\{e^{at}\} = (s - a)^{-1}$$

$$\mathcal{L}\{\sin kt\} = k(s^2 + k^2)^{-1}$$

$$\mathcal{L}\{\cos kt\} = s(s^2 + k^2)^{-1}$$

$$\mathcal{L}\{\sinh kt\} = k(s^2 - k^2)^{-1}$$

$$\mathcal{L}\{\cosh kt\} = s(s^2 - k^2)^{-1}$$

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s - a)$$

$$\mathcal{L}\{f(t - a)U(t - a)\} = e^{-as} F(s)$$

$$\mathcal{L}\{f(t)U(t - a)\} = e^{-as} \mathcal{L}\{f(t + a)\}$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

$$f * g = \int_0^t f(\tau)g(t - \tau)d\tau$$

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$$

$$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = F(s) / s$$

$$\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t)dt, \text{ Period } T.$$

$$\mathcal{L}\{\delta(t - t_0)\} = e^{-st_0}$$

Orthogonal expansions coefficients

Fourier-Bessel coefficients:

For $J_n(\lambda_i b) = 0$,

$$c_i = \frac{2}{b^2 J_{n+1}^2(\lambda_i b)} \int_0^b x J_n(\lambda_i x) f(x) dx$$

For $hJ_n(\lambda_i b) + b\lambda_i J_n'(\lambda_i b) = 0$,

$$c_i = \frac{2\lambda_i^2}{(\lambda_i^2 b^2 - n^2 + h^2) J_n^2(\lambda_i b)} \int_0^b x J_n(\lambda_i x) f(x) dx$$

Fourier coefficients:

$$\int_{-p}^p \cos^2 \frac{n\pi}{p} x dx = \int_{-p}^p \sin^2 \frac{n\pi}{p} x dx = p$$

$$c_n = \frac{1}{2p} \int_{-p}^p f(x) e^{-in\pi x/p} dx$$

Fourier-Legendre coefficients

$$c_n = \frac{2n+1}{2} \int_{-1}^1 f(x) P_n(x) dx$$

Bessel and Legendre Equations

$$x^2 y'' + xy' + (x^2 - \nu^2)y = 0$$

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$

$$(x^n J_n(x))' = x^n J_{n-1}(x)$$

$$(x^{-n} J_n(x))' = -x^n J_{n+1}(x)$$

$$P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1),$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x),$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$