Math 301 Formulas

Green's Theorem: $\oint_C P\,dx + Q\,dy = \iint_R (Q_x - P_y)\,dA$

Stokes' Theorem: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \text{curl}\,\mathbf{F}\cdot\mathbf{n}\,dS$

Divergence Theorem: $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_D \text{div}\mathbf{F}\,dV$

Laplace Transform

$\mathcal{L}\{1\} = s^{-1}$

$\mathcal{L}\{t^n\} = n!/s^{n+1}$

$\mathcal{L}\{e^{at}\} = (s-a)^{-1}$

$\mathcal{L}\{\sin kt\} = k(s^2+k^2)^{-1}$

$\mathcal{L}\{\cos kt\} = s(s^2+k^2)^{-1}$

$\mathcal{L}\{\sinh kt\} = k(s^2-k^2)^{-1}$

$\mathcal{L}\{\cosh kt\} = s(s^2-k^2)^{-1}$

$\mathcal{L}\{f(t)\} = F(s)$

$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$

$\mathcal{L}\{f(t-a)U(t-a)\} = e^{-as}F(s)$

$\mathcal{L}\{f(t)U(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$$

$$f * g = \int_0^t f(\tau)g(t-\tau)\,d\tau$$

$\mathcal{L}\{f*g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = F(s)/s$$

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \text{ Period } T.$$

$\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$

Bessel and Legendre Equations

$x^2y'' + xy' + (x^2-\nu^2)y = 0$

$(1-x^2)y'' - 2xy' + n(n+1)y = 0$

$\left(x^n J_n(x)\right)' = x^n J_{n-1}(x)$

$\left(x^{-n} J_n(x)\right)' = -x^n J_{n+1}(x)$

$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \frac{1}{2}(3x^2-1),$

$P_3(x) = \frac{1}{2}(5x^3-3x),$

$P_4(x) = \frac{1}{8}(35x^4-30x^2+3)$

Orthogonal expansions coefficients

For $J_n(\lambda_i b) = 0$,

$$c_i = \frac{2}{b^2 J_{n+1}^2(\lambda_i b)}\int_0^b xJ_n(\lambda_i x)f(x)\,dx$$

For $hJ_n(\lambda_i b) + b\lambda_i J_n'(\lambda_i b) = 0$,

$$c_i = \frac{2\lambda_i^2}{(\lambda_i^2 b^2 - n^2 + h^2)J_n^2(\lambda_i b)}\int_0^b xJ_n(\lambda_i x)f(x)\,dx$$

$$\int_{-p}^{p}\cos^2\frac{n\pi}{p}x\,dx = \int_{-p}^{p}\sin^2\frac{n\pi}{p}x\,dx = p$$

$$c_n = \frac{1}{2p}\int_{-p}^{p} f(x)e^{-in\pi x/p}\,dx$$

Laplacian

$\Delta u = u_{rr} + r^{-1}u_r + r^{-2}u_{\theta\theta} + u_{zz}$

$$\Delta u = u_{rr} + 2r^{-1}u_r + (r\sin\theta)^{-2}u_{\phi\phi} + r^{-2}u_{\theta\theta} + r^{-2}\cot\theta\, u_\theta$$

Fourier Integral

$$f(x) = \frac{1}{\pi}\int_0^\infty [A(\alpha)\cos\alpha x + B(\alpha)\sin\alpha x]\,dx$$

$$A(\alpha) = \int_{-\infty}^{\infty} f(x)\cos\alpha x\,dx,$$

$$B(\alpha) = \int_{-\infty}^{\infty} f(x)\sin\alpha x\,dx$$

Fourier Transform

$$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{i\alpha x}\,dx = F(\alpha)$$

$$\mathcal{F}^{-1}\{F(\alpha)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\alpha)e^{-i\alpha x}\,d\alpha = f(x)$$

$$\mathcal{F}_s\{f(x)\} = \int_0^\infty f(x)\sin\alpha x\,dx = F(\alpha)$$

$$\mathcal{F}_s^{-1}\{F(\alpha)\} = \frac{2}{\pi}\int_0^\infty F(\alpha)\sin\alpha x\,d\alpha = f(x)$$

$\mathcal{F}\{f''(x)\} = -\alpha^2 F(\alpha)$

$\mathcal{F}_s\{f''(x)\} = -\alpha^2 F(\alpha) + \alpha\, f(0)$

$\mathcal{F}_c\{f''(x)\} = -\alpha^2 F(\alpha) - f'(0)$