KF

Math 260 (022)

Quiz 4 (Ch 4, 5.1)

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Name: ID #: Section #: Serial #:

(2pt) (1) Let $W=\{(x_1,x_2,x_3)\in\mathbb{R}^3 : x_1+x_2+x_3=1\}$. Determine whether or not W is a subspace of $\mathbb{R}^3$.

(2) In the following, determine whether the given vectors are linearly independent or linearly dependent. Justify.

(1pt) a. $\mathbf{v}_1=(2,1), \mathbf{v}_2=(3,2), \mathbf{v}_3=(-1,2)$

(1pt) b. $\mathbf{v}_1=(2,-1,0,3), \mathbf{v}_2=(0,2,0,0)$

(2pt) (3) Determine whether or not $\mathbf{v}_1=(2,1,0), \mathbf{v}_2=(0,2,0), \mathbf{v}_3=(-1,2,1)$ form a basis for $\mathbb{R}^3$.

(2pt) (4) Find the basis for the solution space of

$$\begin{aligned} x_1 \; -3x_2 \; -x_3 &= 0 \\ x_2 \; -3x_3 &= 0 \end{aligned}$$

(2pt) (5) Find the general solution of $y''+2y'-15y=0$.

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1) W is not a subspace as $(1,0,0)$ and $(0,1,0)\in W$

but $(1,0,0)+(0,1,0)=(1,1,0)\notin W$

$\Rightarrow$ W is not closed $\Rightarrow$ it is not a subspace.

2. a) Since $\dim\mathbb{R}^2=2$, then v_1, v_2 and v_3 have to be linearly dependent.

b). $v_1=(2,-1,0,3)$, $v_2=(0,2,0,0)$

$\because$ $v_1\neq\alpha v_2 \;\forall\alpha\in\mathbb{R} \Rightarrow v_1$ and v_2 are linearly independent.

3) $v_1=(2,1,0)$, $v_2=(0,2,0)$

$v_3=(-1,2,1)$

$\because \begin{vmatrix} v_1 & v_2 & v_3 \end{vmatrix} = 4\neq 0$

$\Rightarrow$ $\{v_1, v_2, v_3\}$ forms a basis for $\mathbb{R}^3$

4). The augmented matrix is

$$\left(\begin{array}{ccc|c} 1 & -3 & -1 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right)$$

The leading variables are x_1 and x_2, let $\boxed{x_3=t}$

$\Rightarrow$ $x_2=3t$

and $x_1 = t+3(3t) = 10t$.

$\therefore (x_1,x_2,x_3)=t(10,3,1)$.

$\therefore \left\{\begin{pmatrix}10\\3\\1\end{pmatrix}\right\}$ is a basis for the solution space.

Q5: put $y=\alpha e^{rx}$, substitute:

then $r^2+2r-15=0$

$\Rightarrow$ $(r+5)(r-3)=0$

$\therefore$ The general solution is:

$y(x)=c_1e^{3x}+c_2e^{-5x}$