Math 260 (022)
Exam I

Total Grade: 60
Time: 90 min

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Solve:

(1) $y' = \frac{2e^x}{y}$, $y(0) = 3$.

(2) $xy' = y + xe^{y/x}$.

(3) $\left(1 - \frac{3}{x} + y\right)dx + \left(1 - \frac{3}{y} + x\right)dy = 0$.

(4) $y' + 2xy = x$, $y(0) = 0$.

1) $y' = \frac{2e^x}{y}$, $y(0) = 3$ 7

$\Rightarrow \frac{y}{2} dy = e^x dx$

$\Rightarrow y^2 = e^x + c$

$\because y(0) = 3 \Rightarrow 9 = 1 + c \Rightarrow c = 8$

$\Rightarrow y = \pm\sqrt{e^x + 8}$. But

$y(0) = 3 \Rightarrow y = \sqrt{e^x + 8}$

2) $xy' = y + xe^{y/x}$

$\Rightarrow y' = \frac{y}{x} + e^{y/x}$ (1) 7

let $u = y/x \Rightarrow y = ux$

and $y' = xu' + u$

$\therefore$ Substituting in (1);

$xu' + u = u + e^u$

$\Rightarrow xu' = e^u$ 4

$\Rightarrow e^{-u} du = \frac{1}{x} dx$ 1

$\Rightarrow -e^{-u} = \ln|x| + c$ 1

$\Rightarrow -e^{-y/x} = \ln|x| + c$ 1

3) $\left(1 - \frac{3}{x} + y\right)dx + \left(1 - \frac{3}{y} + x\right)dy = 0$

Ans: This differential equation is exact and

$F_x = \left(1 - \frac{3}{x} + y\right)$, $F_y = \left(1 - \frac{3}{y} + x\right)$

It is exact because

$F_{xy} = F_{yx}$. Then $F(x,y) = c$ is a solution. 7

$\because F_x = \left(1 - \frac{3}{x} + y\right)$

$\Rightarrow F(x,y) = \int\left(1 - \frac{3}{x} + y\right)dx$

$= x - 3\ln|x| + yx + g(y)$

and $F_y = \frac{\partial F}{\partial x} = x + g'(y)$

$= \left(1 - \frac{3}{y} + x\right)$

$\Rightarrow g'(y) = 1 - \frac{3}{y}$

$\Rightarrow g(y) = \int\left(1 - \frac{3}{y}\right)dy$

$= y - 3\ln|y| + c$

$\therefore F(x,y) = x - 3\ln|x| + yx + y - 3\ln|y| + c$

$\therefore x + y(x+1) - 3\ln|xy| = c$

is a general implicit solution.

4) $y' + 2xy = x, \quad y(0) = 0$

$$f(x) = e^{\int^x 2z\,dz} = e^{x^2}$$

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$$\Rightarrow e^{x^2} y' + 2x e^{x^2} y = x e^{x^2}$$

$$\Rightarrow \frac{d}{dx}\left(e^{x^2} y\right) = x e^{x^2}$$

$$\Rightarrow e^{x^2} y = \int x e^{x^2} dx$$

$$= \frac{1}{2} e^{x^2} + c$$

$$\Rightarrow y = \frac{1}{2} + c e^{-x^2}$$

$$\because y(0) = 0 \Rightarrow c = -\frac{1}{2}$$

$$\Rightarrow y = \frac{1}{2} - \frac{1}{2} e^{-x^2}$$

(5) Simplify $y' - y = e^x y^2$ to a linear equation.

Ans: $y' - y = e^x y^2 \quad (1)$

Put $u = y^{-1}$

$\Rightarrow y = u^{-1} \Rightarrow y' = -u^{-2}u'$

$\therefore$ From (1)

$-u^{-2}u' - u^{-1} = e^x u^{-2}$

$\Rightarrow \boxed{u' + u = -e^x}$

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(7) If the echelon of the augmented matrix of a system is $\left[\begin{array}{ccc:c} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & a & b \end{array}\right]$, relate the number of solutions to the values of a and b.

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Ans:

① $a = 0$, no solution when $b \neq 0$
infinitely many soln when $b = 0$

② $a \neq 0$, unique solution for all values of b.

(6) Write the augmented coefficient matrix for the system:

$$\begin{aligned} 2x_1 \quad\quad + x_3 \quad\quad &= 0 \\ x_2 - x_3 \quad\quad &= 1 \\ x_2 \quad\quad + x_4 &= 3 \end{aligned}$$

Ans:

$$\left(\begin{array}{cccc:c} 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 3 \end{array}\right)$$

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(8) Let A and B be matrices of sizes 2×4 and 4×1, respectively. Are the products AB and BA defined? If so, find the size.

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Ans:

AB is defined and $|AB| = 2\times1$

but BA is not defined.

(9) Find the reduced echelon of the matrix

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 3 & -1 & 3 & 4 \\ 6 & -2 & 6 & 8 \end{bmatrix}.$$

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Ans

$$\overset{R_1 \leftrightarrow R_2}{\sim} \begin{bmatrix} 3 & -1 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 6 & -2 & 6 & 8 \end{bmatrix}$$

$$\overset{-2R_1 + R_3}{\sim} \begin{bmatrix} 3 & -1 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\overset{\frac{1}{3}R_1}{\sim} \begin{bmatrix} 1 & -1/3 & 1 & 4/3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\overset{\frac{1}{3}R_2 + R_1}{\sim} \begin{bmatrix} 1 & 0 & \frac{4}{3} & 5/3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$

(10) If $\begin{bmatrix} x & 2 \\ -1 & y \end{bmatrix} = \begin{bmatrix} y & 2 \\ -1 & z \end{bmatrix}$, what is the relation between x and z?

Ans: $x = y$ and $y = z$

$\Rightarrow$ $x = z$

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(11) Let $A = [a_{ij}]$ and $B = [b_{ij}]$. If $3A + B = I$, where I is the 4×4 identity matrix, what is the relation between:

a) a_{21} and b_{21}

b) a_{22} and b_{21}

c) size of A and size of B.

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Ans: a) $3a_{21} + b_{21} = 0$

$\Rightarrow$ $\boxed{b_{21} = -3a_{21}}$

b) No relation.

c) $|A| = |B| = 4 \times 4$.