

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 101
Final Exam
Term 091
Saturday 30/1/2010
Net Time Allowed: 180 minutes

MASTER VERSION

1. If $y = 10^{x^2} + (x^2)^{10}$, then $y'(1) =$
- (a) $20(1 + \ln 10)$
 - (b) $2(10 + \ln 10)$
 - (c) $10(1 + \ln 10)$
 - (d) $10(1 + 2 \ln 10)$
 - (e) $\ln 10$
2. Let $f(x) = \frac{1}{ax + 1}$, where a is a non zero constant. If $f^{(5)}(0) = -(4!)$, then $a^5 =$
- (a) $\frac{1}{5}$
 - (b) $\frac{1}{4}$
 - (c) $\frac{2}{5}$
 - (d) $\frac{1}{3}$
 - (e) $\frac{3}{5}$

3. If c is the number satisfying the conclusion of the Mean Value Theorem for $f(x) = 4 + \sqrt{3x + 1}$ on the interval $[1, 5]$, then $c =$
- (a) $\frac{8}{3}$
 - (b) $\frac{7}{3}$
 - (c) 2
 - (d) $\frac{11}{3}$
 - (e) $-\frac{1}{4}$
4. Which one of the following statements is **TRUE** about the graph of the function $f(x) = x^2 - 18 \ln x$?
- (a) The graph has no inflection points.
 - (b) The graph is concave upward on $(-\infty, -3)$ only.
 - (c) The graph is concave downward on $(3, \infty)$ only.
 - (d) The graph has two inflection points.
 - (e) The graph is concave downward on $(-3, 3)$.

5. The function $f(x) = 3 \sin x - \sin^3 x$, $\frac{\pi}{2} < x < 2\pi$, has
- (a) one critical number.
 - (b) two critical numbers.
 - (c) three critical numbers.
 - (d) four critical numbers.
 - (e) no critical numbers.
6. The graph of the function $f(x) = \frac{x^2 - 3}{x^3}$ is decreasing on the intervals
- (a) $(-\infty, -3)$ and $(3, \infty)$
 - (b) $(-3, 0)$ and $(0, 3)$
 - (c) $(-\infty, -3)$ and $(0, 3)$
 - (d) $(-3, 0)$ and $(3, \infty)$
 - (e) $(-\infty, 0)$ and $(0, \infty)$

7. The asymptotes of the graph of the function $f(x) = \frac{x^4 - x^3 - 2x^2}{x^3 + x^2 + x + 1}$ are
- (a) one slant asymptote and no other asymptotes.
 - (b) one slant, one horizontal, and one vertical asymptotes.
 - (c) one slant and two vertical asymptotes.
 - (d) one slant and three vertical asymptotes.
 - (e) two vertical asymptotes and no other asymptotes.
8. If $x_1 = 2$ is an approximation to one of the real roots of the equation $x^3 + 2x - 13 = 0$, then the next approximation x_2 given by Newton's Method is
- (a) $\frac{29}{14}$
 - (b) $\frac{31}{14}$
 - (c) $\frac{23}{14}$
 - (d) $\frac{15}{7}$
 - (e) $\frac{17}{7}$

9. If $f'(x) = \left(\sqrt{x} - \frac{1}{x}\right)^2$ and $f(4) = \frac{1}{4}$, then $f(1) =$
- (a) -4
 - (b) $\frac{3}{2}$
 - (c) -6
 - (d) $\frac{5}{2}$
 - (e) -7
10. If $y = L$ and $y = M$ are the equations of the horizontal asymptotes to the graph of the function $f(x) = \frac{\pi}{2} - \cos^{-1}\left(\frac{x+1}{\sqrt{4x^2+1}}\right)$, then $L + M =$
- (a) 0
 - (b) $\frac{4\pi}{3}$
 - (c) $\frac{\pi}{6}$
 - (d) $-\frac{\pi}{6}$
 - (e) $-\frac{4\pi}{3}$

11. The equation of the tangent line to the graph of $y = -\pi + 4 \tan^{-1} \left(\frac{2}{x} \right)$ at $x = 2$ is

(a) $y = -x + 2$

(b) $y = -\pi x + 2\pi$

(c) $y = 2x - 4$

(d) $y = \pi x - 2\pi$

(e) $y = -x + \pi$

12. The sum of all values of x for which the function $f(x) = \frac{e^{3x+1}}{e^{4x} - 13e^{2x} + 36}$ is discontinuous, is

(a) $\ln 6$

(b) $\ln 12$

(c) $\ln 20$

(d) $\ln 13$

(e) $\ln 5$

13. If $y = (3x + 1)^{3/2} \sqrt{\frac{2x + 2}{x^2 + 3}}$, then $y'(1) =$
[Hint: you may use logarithmic differentiation]

- (a) 9
- (b) -3
- (c) 11
- (d) 8
- (e) 12

14. The Linearization $L(x)$ of $f(x) = (5 + 3x)^{2/3}$ at $a = 1$ is

- (a) $L(x) = x + 3$
- (b) $L(x) = 2x - 1$
- (c) $L(x) = x - 4$
- (d) $L(x) = \frac{1}{2}x + 2$
- (e) $L(x) = x + 5$

$$15. \quad \left[\cosh\left(\frac{2x}{3}\right) + \sinh\left(\frac{2x}{3}\right) \right]^{3/4} =$$

(a) $\sqrt{e^x}$

(b) $e^{3x/2}$

(c) $4\sqrt{e^x}$

(d) $\frac{1}{2}e^{3x/2}$

(e) $\sqrt{e^x} - \sqrt{e^{-x}}$

$$16. \quad \frac{d}{dx} \left[2x \sinh^{-1}(3x) - \frac{2}{3}\sqrt{1+9x^2} \right] =$$

(a) $2 \sinh^{-1}(3x)$

(b) $\frac{5}{3}\sqrt{1+9x^2} \sinh^{-1}(3x)$

(c) $\frac{4}{3} \sinh^{-1}(3x)$

(d) $6 \sinh^{-1}(3x)$

(e) $6 \sinh^{-1}(3x) + \frac{6x}{\sqrt{1+9x^2}}$

17. $\lim_{x \rightarrow 0} (e^x + 2x)^{3/x} =$

(a) e^9

(b) e^{12}

(c) e^3

(d) 1

(e) ∞

18. $\lim_{x \rightarrow (1/3)^+} \left[\frac{1}{\ln 3x} - \frac{1}{3x - 1} \right] =$

(a) $\frac{1}{2}$

(b) $\frac{1}{6}$

(c) $\frac{2}{3}$

(d) 0

(e) ∞

19. The product of two positive real numbers is 4. If the sum of the square of one of the numbers and the square of twice of the other number is minimum, then the sum of the numbers is

(a) $3\sqrt{2}$

(b) $5\sqrt{2}$

(c) $\frac{3\sqrt{2}}{2}$

(d) $\frac{5\sqrt{2}}{2}$

(e) $2\sqrt{2}$

20. A particle moves in a straight line and has velocity given by $v(t) = \sinh t$. If the initial displacement is $s(0) = \frac{1}{2}$ then $s(2 \ln 2) =$

(a) $\frac{13}{8}$

(b) $\frac{11}{8}$

(c) $\frac{9}{8}$

(d) $\frac{7}{8}$

(e) $\frac{15}{8}$

21. If $x^2 + y^3 = 10$, then the value of $\frac{d^2y}{dx^2}$ at the point $(3, 1)$ is

(a) $-\frac{26}{3}$

(b) $\frac{10}{3}$

(c) -2

(d) $-\frac{5}{6}$

(e) $-\frac{4}{6}$

22. Which one of the following statements is **True** about the function

$$f(x) = -3x^4 + 8x^3 - 10?$$

(a) $f(x)$ has a local maximum and absolute maximum.

(b) $f(x)$ has a local minimum and absolute minimum.

(c) $f(x)$ has a local maximum and no absolute maximum.

(d) $f(x)$ has a local minimum and no absolute minimum.

(e) $f(x)$ has neither absolute maximum nor absolute minimum.

23. A right circular cylinder with largest possible volume is inscribed in a sphere of radius 6. Then the height of such a cylinder is

(a) $4\sqrt{3}$

(b) $3\sqrt{3}$

(c) $\frac{5\sqrt{3}}{2}$

(d) $\frac{7\sqrt{3}}{2}$

(e) $6\sqrt{3}$

24. If the edge of a cube was found to be 30 *cm* with possible error in measurement of 0.1 *cm*, then the estimated percentage error in the surface area of the cube is

(a) $\frac{2}{3}\%$

(b) $\frac{1}{6}\%$

(c) $\frac{3}{2}\%$

(d) $\frac{5}{6}\%$

(e) $\frac{1}{2}\%$

25. The base of an isosceles triangle is fixed at 10 cm . If the two equal sides are increasing at the rate of $\frac{\sqrt{3}}{5}\text{ cm/sec}$, then how fast is the area increasing when the two equal sides are equal to the base?

(a) $2\text{ cm}^2/\text{sec}$.

(b) $\frac{3}{2}\text{ cm}^2/\text{sec}$.

(c) $\frac{5}{2}\text{ cm}^2/\text{sec}$.

(d) $1\text{ cm}^2/\text{sec}$.

(e) $3\text{ cm}^2/\text{sec}$.

26. If M and m are respectively the absolute maximum value and the absolute minimum value of the function

$$f(x) = \begin{cases} x^2 - 4x & \text{if } 0 \leq x < 5 \\ -x + 10 & \text{if } 5 \leq x \leq 6 \end{cases},$$

then $M + m =$

(a) 1

(b) 4

(c) 0

(d) 9

(e) 5

27. If f is a differentiable function, then $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x+2h)}{h} =$

(a) $-f'(x)$

(b) $f'(x)$

(c) 0

(d) $2f'(x)$

(e) $-2f'(x)$

28. Given $f(x) = \frac{(x-1)^3}{x^2}$, $f'(x) = \frac{(x-1)^2(x+2)}{x^3}$, and $f''(x) = \frac{6(x-1)}{x^4}$, which one of the following statements is **False** about f ?

(a) f has no absolute maximum on $(-\infty, 0)$.

(b) $(1, 0)$ is an inflection point.

(c) The graph of f has a slant asymptote.

(d) The graph of f is concave upward on $(1, \infty)$.

(e) The graph of f has a vertical asymptote.