

King Fahd University of Petroleum and Minerals

First Major Exam for Math 202

Time allowed 1 hour and 30 minutes

Full Name:

ID Number:

Section:

Note The following things are prohibited

- Using an advanced calculator
- Having the mobile phone on
- Talking to each other
- Cheating

Major 1

Problem 1 (10 Points) Consider the following DE

$$(1 + x^2)\frac{dy}{dx} + 2xy = \frac{x^2}{1 + x^2}. \quad (1a)$$

- i- Is the DE (1a) linear or nonlinear?
- ii- Write the DE (1a) in the standard form $\frac{dy}{dx} + P(x)y = f(x)$.
- iii- Find the integrating factor $e^{\int P(x)dx}$.
- iv- Solve the DE (1a).

Problem 2 (8 Points) Consider the following DE

$$y\sqrt{1 - x^2}\frac{dy}{dx} = e^{-y}x. \quad (1b)$$

- i- Write the DE (1b) in the form $g(y)dy = f(x)dx$.
- ii- Solve the DE (1b) subject to the initial condition $y(0) = 2$.
- iii- Find the largest interval I on which the solution is defined.

Problem 3 (12 Points) Consider the following DE

$$2y^2dx + x(x - y)dy = 0. \quad (1c)$$

- i- Show that the DE (1c) is homogeneous.
- ii- State all possible types of substitutions that allowed you to transform the DE (1c) into a separable equation.
- iii- Let $y = ux$ where u is a function of x . By substituting y and dy in the DE (1c) show that

$$\frac{u - 1}{u + u^2} du = \frac{dx}{x}. \quad (*)$$

- iv- Solve the DE (*)
- v- Using the above information, find the solution of the DE (1c)

Problem 4 (10 Points) Consider the following IVP

$$y(y + \sin x) dx + (2xy + \ln y^2 - \cos y) dy = 0, \quad y(0) = 1. \quad (1d)$$

- i- Verify that the DE defined in (1d) is exact.
- ii- Solve the IVP (1d).

Note: You may need to use integration by parts in the last part.