

Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Total mark

Table 1: Table of Marks

King Fahd University of Petroleum and Minerals

Second Major Exam for Math 301

Semester 2, Academic year 2007-2008

Time allowed 1 hour and 15 minutes

Full Name:

ID Number:

Section:

Note Important Note

- Show all work
- Having the mobile phone on is prohibited
- Use of programmable calculator is not allowed

Q 1 (4 Points) Find the Laplace transform of the given functions:

a) $f(t) = (e^t - e^{-t})^2$

b) $f(t) = \sin(4t + 5)$

c) $f(t) = e^t * \cos t$

d) $f(t) = te^{-t}\cos 2t$

Solution

Q 2 (4 Points) Find the Laplace inverse of the functions:

a) $F(s) = \frac{s+1}{s^2-4s}$

b) $F(s) = \frac{6}{s^4-9}$

b) $F(s) = \frac{2s^2-3s+4}{s(s+1)(s-2)}$

Solution

Q 3 (8 Points) Use Laplace transform to solve the given initial value problems:

a) $y'' + 9y = e^t$ subject to $y(0) = 0$ and $y'(0) = 1$.

b) $y'' + 4y = \delta(t - 2\pi) + \delta(t - 4\pi)$ subject to $y(0) = 0$ and $y'(0) = 0$ where δ is the dirac delta function.

Note You may need to use the identity: for any nonzero real numbers a and b ,

$$\frac{1}{(s - a)(s^2 + b^2)} = \frac{1}{(a^2 + b^2)(s - a)} - \frac{s + a}{(a^2 + b^2)(s^2 + b^2)}.$$

Solution

Q 4 (4 Points) Use Laplace transform to solve the integral equation:

$$f(t) = \cos t + \int_0^t e^{-s} f(t-s) ds.$$

Solution

Q 5 (5 Points) Consider the function $f(t) = \begin{cases} 1 & -2 \leq x < -1 \\ -x & -1 \leq x < 0 \\ x & 0 \leq x < 1 \\ 1 & 1 \leq x < 2. \end{cases}$

a) Say whether f is an odd or even function.

a) Expand the function f in an appropriate cosine or sine series.

Solution

Q 6 (5 Points)

- a) Find the complex Fourier series of $f(t) = \begin{cases} -1 & -2 \leq x < 0 \\ 1 & 0 \leq x < 2. \end{cases}$
- b) Find the frequency spectrum of the periodic extension of the function $f(t)$.

Solution