

Q 1	Q 2	Q 3	Total mark

Table 1: Table of Marks

King Fahd University of Petroleum and Minerals

First Major Exam for Math 321

Semester 2, Academic year 2007-2008

Time allowed 1 hour and 30 minutes

Full Name:

ID Number:

Section:

Q1- Consider the function $f \in C[a, b]$ with $f(p) = 0$ for some $p \in [a, b]$.

- a- Describe graphically the Newton's method to find an approximation of p .
- b- Show that the Newton's iteration takes the form:

$$p_{k+1} = p_k - \frac{f(p_k)}{f'(p_k)}, \quad k = 0, 1, \dots$$

- c- State the necessary assumptions which guarantee a quadratic convergence of the above sequence $\{p_k\}_{k=0}^{\infty}$ to p .
- d- Let $f(x) = xe^x - 2$. Start with $x_0 = 0.8$, use Newton's iteration to approximate the root of $f(x)$ near 0.8. Stop the iteration when $|f(p_k)| \leq 10^{-6}$.

Q2- Consider the fixed point iteration

$$p_{k+1} = p_k(2 - 7p_k), \quad k = 0, 1, \dots$$

- a- Start from $p_0 = 0.2$, find p_k for $k = 1, 2, 3, 4, 5$.
- b- Find the limit p_* of the sequence $\{p_k\}_{k=0}^{\infty}$ which produced from the above iteration.
- c- Find the order of convergence of the sequence $\{p_k\}_{k=0}^{\infty}$ to p_* .

Q3- Let $f(x) = \sin x - x^2$.

- a- Verify that $f(x) = 0$ has a root r that lies in the interval $[0.8, 0.9]$.
- b- Use four steps of the Bisection method to find an approximation of r .
- c- Determine how many iteration would be required to guarantee that the absolute error between the bisection approximation of r and r itself is $\leq 10^{-6}$.