

Corrigendum to “Class semigroups of integral domains” [J. Algebra 264 (2003) 620-640]

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We are grateful to Paolo Zanardo for pointing out that (i) \Rightarrow (ii) of Theorem 3.2 is not correct. He provided us with the following counterexample. Let V be a one-dimensional valuation domain with maximal ideal M and value group all the real numbers. Then V is not stable since it is not a DVR. But $S(V)$ is Boolean. For, $S(V)$ consists of two constituent groups G_R which is trivial and G_M which is trivial too since $\Gamma(R) = \mathbb{R}$ (Cf. [14, Examples (3), Page 142]).

Indeed, we have located the error in the proof of the implication (i) \Rightarrow (ii) of Lemma 3.5 where we used the fact “ $(V : V_P) = P$ ” which is true only if P is not maximal. Therefore Lemmas 3.6 and 3.7 and Assertion (ii) in Corollary 3.10 should be removed. Accordingly, the statements and proofs of Theorem 3.2 and Lemma 3.5 should read as follows:

Theorem 3.2. *Let R be an integrally closed domain. The following assertions are equivalent:*

- (1) R is a strongly discrete Bezout domain of finite character;
- (2) R is a strongly stable domain.

Moreover, when any one condition holds, R is Boole regular with $\mathcal{S}(R) = \mathcal{F}_{OV}(R)$, where \overline{T} is identified with T for each fractional overring T of R .

Proof. (1) \Rightarrow (2) By [36, Theorem 4.6], R is stable. Now, let I be a nonzero ideal of R and set $T := (I : I)$. Since R is Bezout, then so is T . But I is invertible in T (so finitely generated), then I is principal in T and therefore R is strongly stable. The implication (2) \Rightarrow (1) is handled by a combination of Lemmas 2.1, 3.3, 3.4 and Proposition 2.3. Finally, Boole regularity is proved by (ii) \Rightarrow (i) in the original version. Also the proof of $\mathcal{S}(R) = \mathcal{F}_{OV}(R)$ remains unchanged. \square

Lemma 3.5. *Let V be a valuation domain. The following assertions are equivalent:*

- (1) V_P is a divisorial domain, for each nonzero prime ideal P of R ;
- (2) V is a stable domain;
- (3) V is a strongly discrete valuation domain.

Moreover, when any one condition holds, V is Boole regular.

Proof. The implications (1) \Rightarrow (2) \Leftrightarrow (3) correspond to (ii) \Rightarrow (iii) \Leftrightarrow (iv) in the original proof and are correct. Further, Boole regularity is proved by (iii) \Rightarrow (i). It remains to show (3) \Rightarrow (1). This is clear since for each nonzero prime ideal P of a strongly discrete valuation domain V , $P = PV_P$ is principal. \square

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