Fifth International Fez Conference

Commutative Algebra and Applications

FST Fez–Saiss, University S. M. Ben Abdellah

June 23–28, 2008, Fez, Morocco

Speakers & Abstracts

(Updated on: June 15, 2008)

Contents

On Zariski-like topologies for modules over commutative rings

Jawad Abuhlail
Department of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
Dhahran 31261, Saudi Arabia
abuhlail@kfupm.edu.sa

We consider several primeness (coprimeness) properties of submodules of a given non-zero module over a commutative ring. Assuming suitable conditions, several Zariski topologies are introduced on the spectrum of prime (coprime) submodules in each case.

References

[1] J.Y. Abuhlail, A Zariski topology for bicomodules and corings, Applied Categorical Structures **16** (1-2) (2008) 13–28.

[2] C-P. Lu, The Zariski topology on the prime spectrum of a module, Houston J. Math. **25** (3) (1999) 417–432.

[3] R. McCasland, M. Moore and P. Smith, On the spectrum of a module over a commutative ring, Comm. Algebra **25** (1997) 79–103.

[4] F. Raggi, J. Ríos Montes and R. Wisbauer, Coprime preradicals and modules, J. Pure Appl. Algebra **200** (2005) 51–69.

Induced modules by an endomorphism of finitely generated modules over commutative rings

I. Akharraz
Department of Mathematics, Informatics and Physics
Multidisciplinary Faculty of Taza
BP. 1223 Taza-Garre, Taza, Morroco
i_akharraz@hotmail.com

(Joint work with M. E. Charkani)

Let R be a commutative ring. A finitely generated $R-$module M can be converted into an $R[X]$-module by an R-endomorphism of M (see for example [1]). In this work, we consider endomorphisms which annihilate monic polynomials. Namely, let u be an R-endomorphism of M which annihilates a monic polynomial $f(X)$. Then u converts M into a $R[X]/(f(X))$-finitely generated module M_u. We appeal to our results from [2] to establish a structure theorem for M_u and hence exhibit some of its applications in linear algebra.

References

[1] J. A. Hermida-Alonso and M. Pisoniro, Invariant factors of an endomorphism and finite free resolutions, Linear Algebra and its Applications **187** (1993) 201–226.

[2] I. Akharraz and M. E. Charkani, On the canonical forms for finitely generated modules, Ann. Sci. Math. du Qubec, vol. **28** (1-2) (2004) 13–17.

The techniques of idealization

M. Ali
Department of Mathematics and Statistics
Sultan Qaboos University
Muscat 123, Oman
mali@squ.edu.om

Let R be a commutative ring with identity and M an R-module. The R-module $R(M) = R(+)M$ becomes a commutative ring with identity under the product $(r,m)(r',m') = (rr', r'm + rm')$, called the idealization of M. The idealization of a module is a well-established method to facilitate interaction between a ring on the one hand and a module over a ring on the other. The basic construction is to embed the module M as an ideal in a ring $R(M)$ which contains R as a subring. This technique was used with great success by Nagata. For a comprehensive survey on idealization, Anderson and Winders, Huckaba can be consulted.

In this talk we develop the method of idealization particularly in the context of multiplication, projective, flat and cancelation modules. We show, for example, that if $I(+)N$ is a multiplication (resp. projective, flat, cancellation) ideal of $R(M)$ then I is a multiplication (resp. projective, flat, cancellation) ideal of R. Assuming further that M is multiplication (resp. projective, flat, cancelation) then N is a multiplication (resp. projective, flat, cancelation) submodule of M.

Several results on necessary and sufficient conditions of a homogeneous ideal to be pure, invertible, idempotent, nilpotent, closed, weakly prime, primary, small, large, finitely cogenerated, $1/2$ cancelation, injective or join principal will be considered. We also give necessary and sufficient conditions for $R(M)$ to be a multiplication ring, ZPI-ring, arithmetical ring, (strongly) Prüfer ring, Bezout ring, P-ring, quasi (valuation) ring, coherent ring, finite conductor ring, (Generalized) GCD ring, quasi-Frobenius or distinguished ring.

On pseudo-almost valuation domains

Ayman Badawi
Department of Mathematics and Statistics
American University of Sharjah
P.O. Box 26666, Sharjah, United Arab Emirates
abadawi@aus.edu

Let R be an integral domain with quotient field K and integral closure R'. Anderson and Zafrullah called R an *almost valuation domain* if for every nonzero $x \in K$, there is a positive integer n such that either $x^n \in R$ or $x^{-n} \in R$. In this paper, we introduce a new closely related class of integral domains. We define a prime ideal P of R to be a *pseudo-strongly prime ideal* if, whenever $x, y \in K$ and $xyP \subseteq P$, then there is a positive integer $m \geq 1$ such that either $x^m \in R$ or $y^m P \subseteq P$. If each prime ideal of R is a pseudo-strongly prime ideal, then R is called a *pseudo-almost valuation domain* (*PAVD*). We show that the class of valuation domains, the class of pseudo-valuation domains, the class of almost valuation domains, and the class of almost pseudo-valuation domains are properly contained in the class of pseudo-almost valuation domains; also we show that the class of pseudo-almost valuation domains is properly contained in the class of quasilocal domains with linearly ordered prime ideals. Among the properties of *PAVD*s: we show that an integral domain R is a *PAVD* if and only if for every nonzero $x \in K$, there is a positive integer $n \geq 1$ such that either $x^n \in R$ or $ax^{-n} \in R$ for every nonunit $a \in R$. We show that pseudo-almost valuation domains are precisely the pullbacks of almost valuation domains, we characterize pseudo-almost valuation domains of the form $D + M$, and we use this characterization to construct *PAVD*s that are not almost valuation domains. We show that if R is a Noetherian *PAVD*, then R has Krull dimension at most one and R' is a valuation domain; we show that every overring of a *PAVD* R is a *PAVD* iff R' is a valuation domain and every integral overring of R is a *PAVD*.

Prüfer-like conditions in pullbacks

C. Bakkari
Department of Mathematics
Faculty of Science and Technology
University S. M. Ben Abdellah
Fez, Morocco
cbakkari@hotmail.com

(Joint work with N. Mahdou)

dedicated to Alain Bouvier

We consider five possible extensions of the Prüfer domain notion to the case of commutative rings with zero divisors. We investigate their transfer in pullback constructions in order to generate new classes of examples of rings subject to any given Prüfer-like condition.

References

[1] C. Bakkari and N. Mahdou, Gaussian polynomials and content ideal in pullbacks, Comm. Algebra **34** (8) (2006) 2727–2732.

[2] S. Bazzoni and S. Glaz, Gaussian properties of total rings of quotients, J. Algebra **310** (1) (2007) 180-193.

[3] S. Glaz, The weak dimension of Gaussian rings, Proc. Amer. Math. Soc. **133** (9) (2005) 2507–2513.

[4] S. Glaz, Prüfer conditions in rings with zero-divisors, CRC Press Series of Lectures in Pure Appl. Math. **241** (2005) 272–282.

[5] H. Tsang, Gauss's Lemma, Ph.D. thesis, University of Chicago, Chicago, 1965.

A theorem for numerical semigroups and local rings

V. Barucci
Department of Mathematics
Sapienza, Università di Roma
Piazzale A. Moro 2, 00185 Roma
barucci@mat.uniroma1.it

The talk deals with a theorem which characterizes a certain type of numerical semigroups, the *almost symmetric semigroups*. A numerical semigroup S of maximal ideal $M = S \setminus \{0\}$, of Frobenius number g and canonical ideal $\Omega = \{g - x \mid x \in \mathbb{Z} \setminus S\}$ is almost symmetric if for each $y \in \Omega$, $y + M \subseteq S$. The theorem states that the following conditions are equivalent:

(1) S is almost symmetric.

(2) Each ideal of $M - M = \{z \in \mathbb{Z} \mid z + M \subseteq M\}$ is bidual as ideal of S.

(3) $M - e$ is the canonical ideal of $M - M$, where e is the smallest positive element in S.

An analogous theorem will be given for a local ring R which satisfies appropriate hypotheses. In particular R is supposed to have an m-canonical ideal, i.e. a fractional ideal ω such that $\omega : (\omega : I) = I$, for each regular fractional ideal I of R, [1]. This result in case of one-dimensional Noetherian rings has applications in the study of the canonical model of a singular curve, [2].

The philosophy of the talk is showing as the same idea can be applied in different contexts.

References

[1] W. Heinzer - J. Huckaba - I. Papick, *m-Canonical ideals in integral domains*, Comm. Algebra **26** (1998), 3021–3043.

[2] S.L. Kleiman - R. Vidal Martins, *The canonical model of a singular curve*, preprint 2008.

On modules of bounded projective and flat dimension over commutative rings

S. Bazzoni
Dipartimento di Matematica Pura e Applicata
Università di Padova
Padova, Italy
bazzoni@math.unipd.it

(Joint work with D. Herbera)

We are interested in deciding when the class $\mathscr{P}_n$ of modules of projective dimension at most n, is the direct limit closure of the subclass of the *"compact"* modules. However, as direct limits do not commute with the Ext functor, we investigate the case in which the modules in $\mathscr{P}_n$ are direct summands of modules filtered by the subclass of *"compact"* modules.

By means of conditions on the finitistic dimension of the total quotient ring, we characterize the commutative rings for which every module in $\mathscr{P}_1$ is a direct summand of a module filtered by finitely presented modules belonging to $\mathscr{P}_1$. In particular this will solve completely the problem for integral domains and Noetherian rings.

Concerning the modules of finite flat dimension we will characterize the commutative rings for which the class of modules with flat dimension at most 1 is the direct limit closure of the class of finitely presented modules in $\mathscr{P}_1$, generalizing the famous result by Lazard stating that every flat module is a direct limit of finitely generated projective modules.

On finiteness of chains of intermediary rings

Mabrouk Ben Nasr
Faculty of Sciences of Sfax, Department of Mathematics
3018 Sfax, BP 802, Tunisia
mabrouk_bennasr@yahoo.fr

(Joint work with A. Hamdi)

An extension of integral domains $R \subseteq S$ is said to have the "finite length of intermediate chains of domains" property (for short FICP) if each chain of intermediate rings between R and S is finite. The main purpose of this paper is to characterize when $R \subseteq S$ has FICP in case R^* (the integral closure of R in S) is a finite dimensional semilocal domain. This generalizes a theorem due to Gilmer, in which S is the quotient field of R. Examples illustrating the sharpness and the limits of our results are settled.

n-Perfectness in pullbacks

D. Bennis
Department of Mathematics
Faculty of Science and Technology
University S. M. Ben Abdellah
Fez, Morocco
driss_bennis@hotmail.com

(Joint work with N. Mahdou)

dedicated to Alain Bouvier

A ring is called n-perfect ($n \geq 0$), if every flat module has projective dimension at most n [3]. The n-perfect rings have a homological characterization using the cotorsion global dimension of rings [2], to the effect that R is n-perfect if and only if R has cotorsion global dimension at most n.

This paper continues the investigation of n-perfectness initiated in [1] for pullback constructions. It leads to further examples of n-perfect rings and allows to compute the cotorsion global dimension of some special rings. A result involving flatness in pullbacks is also stated.

References

[1] D. Bennis and N. Mahdou, On n-Perfect Rings and Cotorsion Dimension, submitted for publication. (arXiv:0801.2067v1)

[2] N. Ding and L. Mao, The cotorsion dimension of modules and rings, Abelian groups, rings, modules, and homological algebra, Lect. Notes Pure Appl. Math., **249** (2005) 217–233.

[3] E. E. Enochs, O. M. G. Jenda, and J. A. López-Ramos, Dualizing modules and n-perfect rings, Proc. Edinb. Math. Soc. **48** (2005) 75–90.

Geometric deformations of strong semistability and the localization problem in tight closure theory

Holger Brenner
Fachbereich Mathematik und Informatik
Universität Osnabrück
Albrechtstr. 29, Germany
hbrenner@uni-osnabrueck.de

(Joint work with P. Monsky)

Tight closure is a theory for commutative rings of positive characteristic p which was introduced by Hochster and Huneke twenty years ago and which has many applications in commutative and homological algebra. Technically it assigns to an ideal I in a domain R the ideal (its tight closure)

$$I^* = \{f \in R : \text{ there exists } z \neq 0 \text{ such that } zf^q \in I^{[q]} \text{ for all } q = p^e\}.$$

One of the main open problems was whether tight closure commutes with localization at a multiplicative system.

In this talk we describe an example, based on joint work with Paul Monsky, showing that tight closure does not commute with localization. The example is given by a normal hypersurface domain in dimension three in characteristic two and the ideal is generated by three elements. We consider the ring as a family of graded two-dimensional rings parametrized by the affine line. It turns out that a certain element belongs to the tight closure of the ideal in the generic (transcendental) fiber ring, but never in any special (algebraic) fiber ring. This contradicts the localization property.

The geometry in the background of this example is the existence of a vector bundle on a family of smooth projective curves parametrized by the affine line, such that the bundle on the generic curve is strongly semistable, but not so on any special curve.

Chain conditions (a survey)

P.-J. Cahen
LATP, Faculté des Sciences et Techniques
Université Paul Cézanne
13397, Marseille Cedex 20. France
`paul-jean.cahen@univ-cezanne.fr`

Interesting answers have been recently given by Mabrouk BEN NASR and Noômen JARBOUI to some famous [1] or less famous [1, 2] chain condition conjectures. We propose a short survey on the topic, pointing out the remaining open questions.

References

[1] Ben Nasr, Mabrouk; Jarboui, Noômen, A counterexample for a conjecture about the catenarity of polynomial rings. J. Algebra **248** (2002), no. 2, 785–789.

[2] Jarboui, Noômen, A note on some chain conditions. Arch. Math. (Basel) **90** (2008), no. 2, 133–135.

Integer-valued polynomials on a subset of a valued field

J.-L. Chabert
LAMFA CNRS UMR 6140
Université de Picardie
80039 Amiens, France
jean-luc.chabert@u-picardie.fr

Let K be a field with a rank-one valuation v and let E be any subset of K. The integer-valued polynomials on E are well studied in the case where v is discrete with finite residue field and, more generally, when v is a rank-one valuation and E is a precompact subset of K. Here, we no more assume that E is precompact, and we try to undertake the study of the integer-valued polynomials on E by considering the function $q : \gamma \in \mathbf{R} \mapsto q_\gamma \in \mathbf{N} \cup \{+\infty\}$ where q_γ denotes the number of balls $S(a,\gamma) = \{x \in S \mid v(x-a) \geq \gamma\}$ where $a \in S$. There is a sequence (finite or infinite) of critical valuations $\{\gamma_k\}$ corresponding to the distinct values of q_γ and the number $\gamma_\infty = \sup_k \gamma_k$ play a particular role.

In this general frame, we study several notions such as the polynomial closure of E introduced by McQuillan [3], the characteristic function and the valuative capacity of E, the existence of the v-orderings introduced by Bhargava [1], and a generalization of the notion of regular subset introduced by Y. Amice [1].

References

[1] Y. Amice, Interpolation p-adique, *Bull. Soc. Math. France* **92** (1964) 117–180.

[2] M. Bhargava, P-orderings and polynomial functions on arbitrary subsets of Dedekind rings, *J. reine angew. Math.* **490** (1997), 101–127.

[3] D. L. McQuillan, On a Theorem of R. Gilmer, *J. Number Theory* **39** (1991), 245-250.

Simple hypersurface singularities via totally reflexive modules

L. W. Christensen
Department of Mathematics and Statistics
Texas Tech University
Lubbock, TX 79409, U.S.A.
lars.w.christensen@ttu.edu

(Joint work with G. Piepmeyer, J. Striuli, and R. Takahashi)

Striking connections between the module theory of a local ring and the character of its singularity emerged in the 1980s. A local ring with only finitely many isomorphism classes of indecomposable maximal Cohen-Macaulay modules is said to be of finite CM representation type. By work pioneered by Auslander, every Cohen-Macaulay local ring of finite CM representation type is an isolated singularity. Specialization to Gorenstein rings opens to a finer description: by work of Herzog [2], and of Buchweitz, Greuel, and Schreyer [1], a complete Gorenstein local ring of finite CM representation type is a simple hypersurface singularity.

Let R be a local ring. To avoid the *a priori* condition in [1, 2] that R is Gorenstein, we replace finite CM representation type with a finiteness condition on the category of totally reflexive R-modules (modules of Gorenstein dimension 0 in the sense of Auslander and Bridger). We prove that if the set of isomorphism classes of indecomposable totally reflexive R-modules is finite, then either it has exactly one element, represented by R itself, or R is Gorenstein and an isolated singularity—if R is complete, then it is even a simple hypersurface singularity.

References

[1] R.-O. Buchweitz, G.-M. Greuel, and F.-O. Schreyer, *Cohen-Macaulay modules on hypersurface singularities II*, Invent. Math. **88** (1987), 165–182.

[2] J. Herzog, *Ringe mit nur endlich vielen Isomorphieklassen von maximalen, unzerlegbaren Cohen-Macaulay-Moduln*, Math. Ann. **233** (1978), 21–34.

Almost clean rings and arithmetic rings

François Couchot
Laboratoire de Mathématiques Nicolas Oresme, CNRS UMR 6139,
Département de mathématiques et mécanique
Université de Caen Basse-Normandie
14032 Caen cedex, France
couchot@math.unicaen.fr

In this talk we will show that a commutative Bézout ring R with compact minimal prime spectrum is an elementary divisor ring if and only if so is R/L for each minimal prime ideal L. This result is obtained by using the quotient space pSpec R of the prime spectrum of the ring R modulo the equivalence generated by the inclusion. When every prime ideal contains only one minimal prime, for instance if R is arithmetic, pSpec R is Hausdorff and there is a bijection between this quotient space and the minimal prime spectrum Min R, which is a homeomorphism if and only if Min R is compact. If x is a closed point of pSpec R, there is a pure ideal $A(x)$ such that $x = V(A(x))$. If R is almost clean, i.e. each element is the sum of a regular element with an idempotent, we will show that pSpec R is Hausdorff and totally disconnected and, $\forall x \in$ pSpec R, $R/A(x)$ is almost clean; the converse holds if every principal ideal is finitely presented. A question posed by Facchini and Faith at the second International Fez Conference on Commutative Ring Theory in 1995, will be also investigated. If R is a commutative ring for which the ring $Q(R/A)$ of quotients of R/A is an IF-ring for each proper ideal A, we will prove that R_P is a strongly discrete valuation ring for each maximal ideal P and R/A is semicoherent for each proper ideal A.

Semigroups of valuations dominating Noetherian domains

Steven Dale Cutkosky
Department of Mathematics
University of Missouri
202 Mathematical Sciences Bldg, University of Missouri,
Columbia, MO 65211 USA
cutkoskys@missouri.edu

Suppose that A is a Noetherian domain, and ν is a valuation of the quotient field of A which dominates A, with value group $\Gamma(\nu)$. $\Gamma(\nu)$ is an ordered subgroup of $\mathbf{R}^d$ with the lex order, where d is the dimension of A. The groups $\Gamma(\nu)$ which are attainable by such ν have been classified by MacLane and Schilling, Zariski and Samuel, and Kuhlmann. However, the possible semigroups $S^R(\nu)$ of values $\nu(f)$ of nonzero elements of the maximal ideal of R are not well understood. In Zariski and Samuel's book "Commutative Algebra", necessary conditions are given for a semigroup to be a semigroup $S^R(\nu)$ of a valuation ν dominating a Noetherian domain R. The restrictions are that the rational rank must be less than or equal to the dimension d of R, and that $S^R(\nu)$ must be well ordered, of ordinal type ω^h, for some $h \leq d$.

In joint work with Bernard Teissier, we give some examples indicating that there seem to be few restrictions on a semigroup being a valuation semigroup dominating some noetherian domain, besides the two listed above. We show that higher rank semigroups can be particularly chaotic. In collaboration with Kia Dalili and Olga Kascheyeva, we show that the semigroups of rank 1 valuations have bounded growth, which allows us to give very simple examples of subsemigroups of the positive rational numbers which are well ordered of ordinal type ω, but are not the valuation semigroup of a valuation ν dominating any noetherian domain, showing that the necessary conditions of Zariski and Samuel are not sufficient. We also make a study of realizable growths of rank 1 semigroups $S^R(\nu)$. In further work with Bernard Teissier, we obtain polynomial bounds for the growth of higher rank valuations.

On the Buchsbaumness of the associated graded ring of semigroup rings

M. D'Anna
Department of Mathematics and Computer Science
Università degli studi di Catania
Catania, Italy
`mdanna@dmi.unict.it`

(Joint work with M. Mezzasalma and V. Micale)

Let $R = k[[t^{g_1}, \dots, t^{g_n}]]$ be the semigroup ring associated to the numerical semigroup $S = \langle g_1, \dots, g_n \rangle$. We study when its associated ring $G(\mathfrak{m})$ is Buchsbaum; in particular, we give a theoretical characterization for $G(\mathfrak{m})$ to be Buchsbaum, not Cohen-Macaulay; moreover we introduce some families of invariants for S, connected with the Apery sets of S and of its blow-up S' and we use them in order to give a necessary and a sufficient condition for $G(\mathfrak{m})$ to be Buchsbaum.

References

[1] M. D'Anna, M. Mezzasalma and V. Micale, *On the buchsbaumness of the associated graded ring of a one-dimensional local ring*, Comm. Algebra (to appear)

Modules admit Cousin complexes with finite cohomologies

Mohammad T. Dibaei
Faculty of Mathematical Sciences
Tarbiat Moallem University
Tehran, Iran
dibaeimt@ipm.ir

(Joint work with R. Jafari)

Cousin complexes were introduced by Hartshorne (1966) in algebraic geometry and then its commutative algebra analogue is given by Sharp (1969). It is well known that a finitely generated module M over a commutative Noetherian ring A is Cohen-Macaulay if and only if its Cousin complex, $\mathscr{C}_A(M)$, is exact. Therefore the class of finitely generated A-modules whose Cousin complexes have finitely generated cohomologies may be considered as a generalization of Cohen-Macaulay modules. The study of this class has been started by Dibaei and Tousi [2] and continued with Dibaei [1] and Kawasaki [3]. Also Lipman, Nayak, and Sastry (2005, 2007) studied the idea in a more general situation in algebraic geometry.

In this talk, I discuss a characterization of the above class in terms of uniform local cohomological annihilators.

References

[1] M. T. Dibaei, A study of Cousin complexes through the dualizing complexes, Comm. Alg. **33** (2005) 119–132.

[2] M. T. Dibaei and M. Tousi, A generalization of the dualizing complex structure and its applications, J. Pure Appl. Algebra **155** (2001) 17–28.

[3] T. Kawasaki, Finiteness of Cousin cohomologies, Trans. Amer. Math. Soc. **360** (2008) 2709–2739.

Generic fiber of power series ring extensions

Tiberiu Dumitrescu
Faculty of Mathematics and Informatics
University of Bucharest
Academiei Str. 14, Bucharest, RO-010014, Romania
tiberiu_dumitrescu2003@yahoo.com, tiberiu@fmi.unibuc.ro

An extension $A \subseteq B$ of integral domains is said to be a *trivial generic fiber* (TGF) extension, if every nonzero prime ideal of B has nonzero intersection with A. Let K be a field and $z, x_1, x_2, ..., x_n$ indeterminates over K. In [1], Heinzer, Rotthaus and Wiegand proved that the mixed polynomial/power series ring extension

$$K[x_1, x_2, ..., x_n][[z]] \hookrightarrow K[x_1, x_2, ..., x_n, 1/x_1][[z]]$$

is TGF for $n = 1$ and non-TGF for $n = 3$.

In this talk, we extend the result above proving the following theorem. Let D be a Noetherian domain containing a field, $a \in D$ a nonzero nonunit and z an indeterminate over D. Then the generic fiber of the extension $D[[z]] \hookrightarrow D[1/a][[z]]$ has dimension $\geq dim(D/aD)$.

References

[1] W. Heinzer, C. Rotthaus and S. Wiegand, Mixed polynomial/power series and relations among their spectra. In *Multiplicative Ideal Theory in Commutative Algebra A Tribute to the Work of Robert Gilmer*, pp. 227-242. Ed. J. Brewer, S. Glaz, W. Heinzer and B. Olberding. Springer-Verlag, New York, 2006.

On the Hochster dual of a topological space

O. Echi
Department of Mathematics
University of Tunis el-Manar
Tunis 2092, Tunisia
othechi@yahoo.com

(Joint work with R. Gargouri and S. Lazaar)

This paper deals with a dual topology introduced by Hochster; this duality (in the sense of Kopperman [5]) is applied to two particular classes of spaces related to spectral spaces (namely, down-spectral spaces and up-spectral spaces). These two classes are recently introduced by Echi and Belaid in [1]. Dual topological properties are also introduced and studied.

References

[1] K. Belaid, O. Echi, On a conjecture about spectral sets, Topology Appl. **139** (2004) 1–15.

[2] J. deGroot, H. Herrlich, G. E. Strecker and E. Wattel, Compactness as an operator, Compositio. Math. **21** (1969) 349–375.

[3] J. deGroot, An isomorphism principal in general topology, Bull. Amer. Math. Soc. **73** (1967) 465–467.

[4] M. Hochster, Prime ideal structure in commutative rings, Trans. Amer. Math. Soc. **142** (1969) 43–60.

[5] R. Kopperman, Asymmetry and duality in topology, Topology Appl. **66** (1995) 1–39.

Computational of an integral basis of a quartic number field

L. El Fadil
Multidisciplinary Faculty of Ouarzazate
FPO, P.O. 638 Ouarzazate-Morocco
lhouelfadil@hotmail.com

In this paper, for each prime integer p, a p-integral basis of a quartic number field $K = \mathbb{Q}(\alpha)$ is given, where α is a root of an irreducible polynomial $P(X) = X^4 + aX + b \in \mathbb{Z}[X]$. The discriminant d_K of K and an integral basis of K are then obtained from its p-integral bases. A necessarily and sufficient condition for the existence of a power integral basis is given.

References

[1] H. Cohen, A course in computational algebraic number theory, GTM 138, Springer-Verlag, Berlin, 1995.

[2] L. El Fadil, Computational of the integral closure. (To appear in International Journal of Commutative Rings)

[3] L. El Fadil, Computational of a power integral basis of a pure cubic number field, I.J.C.M.S **2** (2007) 13–16.

[4] J. Montes and N. Enric, J. Algebra **146** (1992) 318–334.

Self-distributive groupoids from an algebraic point of view

M. Elhamdadi
Department of Mathematics
University of South Florida
Tampa, FL 33620, USA
emohamed@math.usf.edu

(Joint work with Scott Carter, Alissa Crans, and Masahico Saito)

We will introduce an algebraic structure called self-distributive groupoid (sometimes called also quandle). Its axiomatization comes from knot theory. Quandles can be seen as an abstraction from groups since many important examples arise as conjugation invariant subsets of groups. Quandles can be used, for example, to give invariant of knots and knotted surfaces and also to study Nichols algebras. In many situations we are faced with the following questions: Determine the general structure of finite quandles, compute their cohomologies in particular the second and third cohomology groups.

The talk will contain enough examples and calculations.

Integer-valued polynomial rings and t-linked extensions

Jesse Elliott
Department of Mathematics
California State University, Channel Islands
Camarillo, CA 93012, USA
jesse.elliott@csuci.edu

Using the tools of t-ideals and associated primes, we generalize some known results on integer-valued polynomial rings over Krull domains. These tools are useful for studying the ring $\mathrm{Int}(D)$ of integer-valued polynomial rings over an integral domain D primarily because if $\mathfrak{p}$ is any prime ideal of D such that $\mathrm{Int}(D_\mathfrak{p}) \neq D_\mathfrak{p}[X]$, then $\mathfrak{p}$ is a weak Bourbaki associated prime, hence a t-ideal, of D with finite residue field. Our main result is as follows. Let D be an integral domain such that, for every t-maximal ideal $\mathfrak{p}$ of D with finite residue field, the ideal $\mathfrak{p}D_\mathfrak{p} \cap R$ is principal in some polynomially dense subring R of $D_\mathfrak{p}$, and $\mathrm{Int}(D_\mathfrak{p}) = \mathrm{Int}(D)_\mathfrak{p}$. For instance, D could be any TV PVMD. Then (1) the domain $\mathrm{Int}(D)$ of integer-valued polynomials on D is locally free as a D-module; (2) $\mathrm{Int}(S^{-1}D) = S^{-1}\,\mathrm{Int}(D)$ for any multiplicative subset S of D; (3) for every positive integer n the multivariable integer-valued polynomial ring $\mathrm{Int}(D^n)$ decomposes as an n-fold tensor product over D of $\mathrm{Int}(D)$; and (4) for any t-linked extension A of D, the domain $\mathrm{Int}(D,A)$ is generated by $\mathrm{Int}(D)$ as an A-module, and A contains D as a polynomially dense subring if and only if A is unramified and has trivial residue field extensions at every t-maximal ideal of D with finite residue field.

Properties of lexsegment ideals

Viviana Ene
Faculty of Mathematics and Computer Science
Ovidius University of Constanta
Bd. Mamaia 124, 900527 Constanta, Romania
`vivian@univ-ovidius.ro`

(Joint work with Anda Olteanu and Loredana Sorrenti)

Let $\mathscr{M}_d$ be the set of all monomials of degree d in the polynomial ring $S = K[x_1, \ldots, x_n]$ over a field K, ordered lexicographically by $x_1 > \ldots > x_n$. A *lexsegment ideal* of degree d is a subset of $\mathscr{M}_d$ of the form $\mathscr{L}(u,v) = \{w \in \mathscr{M}_d \mid v \leq w \leq u\}$ for some $u, v \in \mathscr{M}_d,\ v \leq u$. A *lexsegment ideal* is an ideal of S generated by a lexsegment. Lexsegment ideals with linear resolutions have been characterized in [1]. We show that all lexsegment ideals with linear resolutions have linear quotients with respect to a suitable ordering of the generators.

A lexsegment $\mathscr{L}(u,v)$ is called *completely* if all its iterated shadows are again lexsegments. For complete lexsegment ideals with linear resolution one can give an explicit minimal free resolution by using the mapping cone construction [2].

Finally we study the depth and the dimension of lexsegment ideals. Our results show that one may compute these invariants just looking at the ends of the lexsegment. As an application, we characterize the Cohen-Macaulay lexsegment ideals.

We acknowledge the support provided by the Computer Algebra Systems CoCoA and SINGULAR for the extensive experiments which helped us to obtain some of the results of this work.

References

[1] A. Aramova, E. De Negri, J. Herzog, Lexsegment ideals with linear resolutions, Illinois J. Math., **42**(3), 1998, 509–523.

[2] J. Herzog, Y. Takayama, Resolutions by mapping cone, Homology Homotopy Appl., **4**, no. 2, 2002, 277–294.

Integer valued polynomials in several variables : v-orderings and bases

S. Evrard
LAMFA CNRS UMR 6140 , IUFM
Université de Picardie,
80039 Amiens, France
sabine.evrard@u-picardie.fr

Let E be a subset of a ring D with quotient field K. The set $\mathrm{Int}(E,D)$ of integer valued polynomials on E is the set of polynomials of $K[X]$ which send E into D. Denoting by $\mathfrak{I}_n(E)$ the set formed by all the coefficients of all the polynomials of $\mathrm{Int}(E,D)$ with degree $\leq n$, we know that $\mathfrak{I}_n(E)$ is a fractional ideal of A as soon as E is infinite. Moreover $\mathrm{Int}(E,D)$ has a regular basis if and only if , $\forall n$, $\mathfrak{I}_n(E)$ is a principal fractional ideal of D [2]. When $D = V$ is the ring of a discrete valuation v, Bhargava uses sequences of E called v-orderings [1] to construct some regular bases of $\mathrm{Int}(E,V)$. When $E \subseteq V^d$ $(d \geq 1)$, $\mathrm{Int}(E,V)$ is the set formed by the polynomials of $K[X_1,\ldots,X_d]$ which send E into V. We still denote by $\mathfrak{I}_n(E)$ the set of all the coefficients of all the polynomials of $\mathrm{Int}(E,V)$ with (total) degree $\leq n$. We first recall conditions on E such that the $\mathfrak{I}_n(E)$ are fractional ideals of V, and distinct from V [1]. Then, we introduce the Vandermonde ideals of E and we define the notion of v-ordering of E. Finally, we describe bases of $\mathrm{Int}(E,V)$ associated to these v-orderings. This allows us to extend the notion of factorials with respect to the subset E.

References

[1] M. Bhargava, *P*-orderings and polynomial functions on arbitrary subsets of Dedekind rings, J. Reine Angew. Math. **490** (1997) 101–127.

[2] P.J. Cahen, J.L. Chabert, Integer-valued polynomials, Mathematical Surveys and Monographs, vol **48**, Amer. Math. Soc., Providence 1997.

[3] S.B. Mulay, Integer Valued Polynomials in Several Variables, Comm. Algebra **27** (5) (1999) 2409–2423.

v-Ordering sequences and countable sets

Y. Fares
LAMFA CNRS UMR 6140 , IUFM
Université de Picardie,
80039 Amiens, France
youssef.fares@u-picardie.fr

The notion of v-ordering for a subset E of the domain V of a valuation v introduced by M. Bhargava turns out to be very useful for the construction either of bases of the V-module Int(E,V) of integer-valued polynomials on E, or of normal bases of the ring C(E,V) of continuous functions from E to V. The aim of this paper is to show that, when E is a countable and precompact subset of V, one can construct a sequence formed by exactly all the elements of E which is a v-ordering of E. In order to do this, we define a polynomial topology associated with the polynomial equivalence introduced by D. MacQuillan.

Generic bounds for tight closure and Frobenius closure

H. Fischbacher-Weitz
Fachbereich Mathematik/ Informatik
Universität Osnabrück
Albrechtstr. 28a
49069 Osnabrück, Germany
hfischba@mathematik.uni-osnabrueck.de

(Joint work with H. Brenner)

Let k be a field of positive characteristic. We use a geometric method to show that given a degree bound for ideal membership in the polynomial ring $P = k[x_1,\ldots,x_t]$, one obtains a generic degree bound for membership in the tight closure of any R_+-primary ideal in a standard-graded k-algebra R of dimension t. Moreover, if R is normal, one also obtains a generic bound for membership in the Frobenius closure. In low dimensions, the bound for ideal membership in $k[x_1,\ldots,x_t]$ can be computed from the known cases of the Fröberg conjecture.

On v-domains: a survey

Marco Fontana
Dipartimento di Matematica
Università degli Studi "Roma Tre"
00146 Roma, Italy
fontana@mat.uniroma3.it

(Joint work with **Muhammad Zafrullah**)

dedicated to Alain Bouvier
for the long-standing collaboration and friendship

An integral domain D is a v–domain if, for every finitely generated nonzero (fractional) ideal F of D, we have $(FF^{-1})^{-1} = D$. The v–domains generalize Krull domains, Prüfer domains and other classes of Prüfer-like domains and have appeared in the literature with different names. This survey is the result of an effort to put together information on this useful class of integral domains. In the talk, I will try to present old, recent and new characterizations of v–domains along with some historical remarks. I will also discuss the relationship of v–domains with some of their various specializations and generalizations, giving suitable examples.

Non-unique factorization of polynomials over finite rings

S. Frisch
Institut für Mathematik A
Technische Universität Graz
Steyrergasse 30, A-8010 Graz, Austria
`frisch@tugraz.at`

(Joint work with Ch. Frei)

We investigate the non-uniqueness of factorization into irreducibles of polynomials over finite rings. We show that different ways of defining irreducible elements in rings with zero-divisors coincide in the polynomial ring over a sufficiently nice finite ring, such as $\mathbb{Z}_{p^n}$, or, more generally $R[x]$, where R is a Galois ring.

The elasticity of the multiplicative monoid of $R[x]$ is infinite, and what is more, ρ_2 is infinite, meaning that there are examples of a product of 2 irreducibles also factoring as a product of m irreducibles, for arbitrarily large m.

w-Divisoriality in polynomial rings

S. Gabelli
Dipartimento di Matematica
Università degli Studi Roma Tre
00146 Roma, Italy
gabelli@mat.uniroma3.it

(Joint work with E. Houston and G. Picozza)

A domain D is said to be *divisorial* if each of its nonzero ideals is divisorial. These domains have been studied by Bass [1], Matlis [6], Heinzer [5], Bazzoni and Salce [2], and Bazzoni [3]. In [4], joint work with S. El Baghdadi, we studied domains in which each w-ideal of D is divisorial and dubbed such domains *w-divisorial* domains. In part this work is a sequel to that paper.

Our main goal is to study whether w-divisoriality transfers from D to the polynomial ring $D[X]$. Although we do not give a definitive answer, we do show that the property transfers in many cases, and we analyze the difficulties in the general case.

References

[1] H. Bass, On the ubiquity of Gorenstein rings, Math. Z. **82** (1963) 8–28.

[2] S. Bazzoni and L. Salce, Warfield domains, J. Algebra **185** (1996) 836–868.

[3] S. Bazzoni, Divisorial domains, Forum Math. **12** (2000) 397–419.

[4] S. El Baghdadi and S. Gabelli, w-Divisorial domains, J. Algebra **285** (2005) 335–355.

[5] W. Heinzer, Integral domains in which each non-zero ideal is divisorial, Mathematika **15** (1968) 164–170.

[6] E. Matlis, Reflexive domains, J. Algebra **8** (1968) 1–33.

Gaussian properties and ring constructions

Sarah Glaz
Department of Mathematics
University of Connecticut
Storrs, CT 06269, USA
glaz@math.uconn.edu

Let R be a commutative ring and let f be a polynomial with coefficients in R. Denote by $c(f)$, *the content* of f, the ideal of R generated by the coefficients of f. A ring R is called a *Gaussian ring* if $c(f)c(g) = c(fg)$ for any two polynomials f and g with coefficients in R. Gaussian rings were defined by Tsang in 1965 and became an active topic of investigation due to their connection to Kaplansky's conjecture, which was solved between 1997 and 2005. The focus of these investigations lies in the comparison between the Gaussian property and several other ring theoretic and homological properties. Specifically the properties under consideration are:

(1) R is a semihereditary ring.
(2) $w.\dim R \leqslant 1$
(3) R is an arithmetical ring.
(4) R is a Gaussian ring.
(5) R is a Prüfer ring.

If R is an integral domain all five properties coincide with the Prüfer domain notion. For general rings the situation is more complicated and investigating it took a number of paths and approaches.

This talk will survey the results obtained by the speaker and other authors comparing the five properties in a general setting or under restrictions imposed on the total ring of quotients of the rings involved. We will also survey recent work done by others investigating the behavior of these properties in pullback ring constructions. We will conclude with an overview of the speaker's work in progress on the behavior of these properties in commutative group rings.

Strong exceptional sequences of vector bundles on certain Fano varieties

M. Halic
Department of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
Dhahran 31261, Saudi Arabia
halic@kfupm.edu.sa

Exceptional sequences of vector bundles/sheaves over a variety X are special generators of the additive category $D^b(CohX)$. Kapranov proved the existence of tilting bundles over homogeneous varieties. King conjectured in [Ki97] the existence of tilting sequences of vector bundles on projective varieties which are obtained as quotients of Zariski open subsets of affine spaces.

Although the conjecture does not hold in general, it remains the problem of constructing examples of varieties admitting tilting bundles. For toric varieties, examples of exceptional bundles have been given by Altmann and Hille in [AH99], and by Costa and Miró-Roig in [CM-R04].

The goal of my talk is to give further examples of projective varieties carrying exceptional sequences of vector bundles. The varieties are obtained as geometric invariant quotients of affine spaces by linear actions of reductive groups, as in King's conjecture.

References

[AH99] *K. Altmann, L. Hille*, Strong exceptional sequences provided by quivers, Algebr. Represent. Theory **2** (1999) 1-17.

[CM-R04] *L. Costa, R.M. Miró-Roig*, Tilting sheaves on toric varieties, Math. Z. **248** (2004) 849-865.

[Ki97] *A. King*, Tilting bundles on some rational surfaces, preprint available at: http://www.maths.bath.ac.uk/∼masadk/papers/.

A general theory of invertible ideals and application to Prüfer-like monoids and domains

F. Halter-Koch
Institute for Mathematics and Scientific Computing
Karl-Franzens-Universität Graz
Heinrichstraße 36, A-8010 Graz, Austria
franz.halterkoch@aon.at

Let r be an ideal system (a star operation) on a cancellative monoid or a domain D. Then D is called r-Prüfer if every r-finite r-ideal is r-invertible. First we discuss relative versions of the concept of r-invertibility (motivated by a recent manuscript of D.D.Anderson, D.F.Anderson, M.Fontana and M.Zafrullah) and apply it to characterize Prüfer-like monoids and domains. Then we prove a generalized version of Bazzoni's conjecture: If r is finitary, then an r-Prüfer monoid is of Krull type if every r-locally principal r-ideal is r-invertible. We apply this result to characterize monoids and domains whose r-ideal (class) semigroup is Clifford.

References

[1] F. Halter-Koch, Clifford semigroups of ideals in monoids and domains. Forum Math., to appear.

Chain conditions in rings of the form $A+(X_1,\dots,X_n)B[X_1,\dots,X_n]$

S. Hizem
Département de Mathématiques
Faculté des Sciences de Monastir
Monastir, Tunisia
hizems@yahoo.fr

Let $A \subset B$ be an extension of commutative rings with unity and $X_1,\dots,X_n$ a set of indeterminates over B. In this talk we study some chain conditions in pullbacks of the form $R = A+(X_1,\dots,X_n)B[X_1,\dots,X_n]$. We show that R is Noetherian if and only if A is Noetherian and B is a finitely generated A-module, and that $\mathrm{Spec}(R)$ is Noetherian if and only if $\mathrm{Spec}(A)$ and $\mathrm{Spec}(B)$ are Noetherian. We also investigate necessary and sufficient conditions for the ring R to be strongly Laskerian, Laskerian or ZD.

References

[1] I. Armeanu, On a class of Laskerian rings, Roum. Math. Pures et Appl. XXII, 8, 1033–1036, Bucarest, 1977.

[2] V. Barucci and M. Fontana, When are D+M rings Laskerian, Rendiconti del Circlo Math Di Palermo, Serie II, XXXI (1982), 125–136.

[3] M. Fontana, Topologically defined classes of commutative rings, Ann. Mat. Pura. Appl. 123 (1980), 331–355.

[4] M. Fontana, L. Izelgue, and S. Kabbaj, Quelques propriétés des chaînes d'idéaux dans les anneaux $A+XB[X]$, Comm. Algebra 22 (1) (1994), 9–27 .

[5] H. A. Hussain, A class of Laskerian rings which do not satisfy the altitude theorem of Krull, Roum. Math. Pures et Appl. XXV, 1, 43–49, Bucarest, 1980.

André-Quillen homology and classes of arithmetic rings

C. Ionescu
Institute of Mathematics *Simion Stoilow* of the Romanian Academy
PO Box 1-764, Bucarest, Romania
Cristodor.Ionescu@imar.ro

(Joint work with T. Dumitrescu)

We compute the first André-Quillen homology modules for the simple overrings of integrally closed domains and we study an ideal-theoretic condition arising from the vanishing of H_1.

On the prime ideal structure of Bhargava rings

L. Izelgue
Department of Mathematics
Faculty of Sciences Semlalia
Cadi Ayyad University
Marrakech 40000, Morocco
izelgue@ucam.ac.ma

(Joint work with I. Al-Rasasi)

dedicated to Alain Bouvier

Let D be a domain with quotient field K. We denote by $\mathrm{Int}(D)$ the ring of integer-valued polynomials, i.e., $\mathrm{Int}(D) := \{f \in K[X] \mid f(D) \subseteq D\}$ [2]. For each $x \in D$, $x \neq 0$, we consider the ring $\mathbb{B}_x(D) = \{f \in K[X] \mid \forall a \in D, f(xX+a) \in D[X]\}$, called Bhargava ring [3]. We have $\mathbb{B}_x(D) = \bigcap_{a \in D} D[\frac{X-a}{x}]$ and $\mathrm{Int}(D) = \bigcup_{0 \neq x \in D} \mathbb{B}_x(D)$ [3]. The fact that for each unit $u \in D$, $\mathbb{B}_u(D) = D[X]$ leads us to first classify the elements $x \in D$ such that $\mathbb{B}_x(D) \neq D[X]$. We then give a detailed description of the prime spectrum of $\mathbb{B}_x(D)$. Also, we shed some light on the problem of lifting prime ideals of $D[X]$ to $\mathbb{B}_x(D)$. In this context, we determine conditions under which the extension ideal $p[X] \in \mathrm{Spec}(D[X])$ is lost in $\mathbb{B}_x(D)$. As a consequence of these investigations, we give an approximation to $\dim \mathbb{B}_x(D)$. We then compute the valuative dimension of $\mathbb{B}_x(D)$ and give necessary and sufficient conditions for $\mathbb{B}_x(D)$ to be a Jaffard domain [1].

References

[1] Anderson, D. F., Bouvoier, A., Dobbs, D. E., Fontana, M., and Kabbaj, S., On Jaffard Domains, Expo. Math. 6 (1988), 145-175.

[2] Cahen, P.-J. and Chabert, J.-L., *Integer–Valued Polynomials*, AMS Surveys and Monographs 48, Providence, 1997.

[3] Yeramian, J., *Anneaux de Bhargava*, Ph.D. Thèse, Univerité de Aix-Marseille III, Marseille, France, 2004.

Universally catenarian domains of the type $A+I$

Noomen Jarboui
Department of Mathematics
King Faisal University
Al-Ahsa 31982, Saudi Arabia.
noomenjarboui@yahoo.fr

(Joint work with Ahmed Ayache)

We deal with subrings of the type $R = A + I$ of a domain T of $D + I$ type where D is a domain and I is a nonzero prime ideal of T, generalizing the classical $A + M$ construction. We examine the possible transfer of the universal catenarity from A and T to R. This study allows us to generalize and improve some known results and to provide several interesting applications and examples.

On (n,d)-perfect rings

A. Jhilal
Department of Mathematics
Faculty of Science and Technology
University S. M. Ben Abdellah
Fez, Morocco
Jhilalabdo@hotmail.com

(Joint work with N. Mahdou)

dedicated to Alain Bouvier

In this paper, we introduce the notion of (n,d)-perfect ring which is, in some way, a generalization of the notion of $A(n)$ ring. We state some basic results and survey the relationship between the $A(n)$ and (n,d)-perfect properties. We also investigate the (n,d)-perfect property in pullback rings.

References

[1] N. Bourbaki, Algèbre commutative, Chapitres 1-4, Masson, Paris, 1985.

[2] H. Cartan and S. Eilenberg, Homological algebra, Princeton University press, 1956.

[3] S. H. Cox, Jr. and R. L. Pendleton, Rings for which certain flat modules are projective, Trans. Amer. Math. Soc. **150** (1970) 139–156.

[4] D. E. Dobbs, S. Kabbaj, N. Mahdou, and M. Sobrani, When is $D+M$ n-coherent and an (n,d)-domain?, Lect. Notes Pure Appl. Math. Dekker **250** (1999) 257–270.

[5] S. Glaz; Commutative coherent rings, Springer-Verlag, Lecture Notes in Mathematics, 1371, 1989.

Constituent groups of Clifford semigroups arising from t-closure

S. Kabbaj
Department of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
Dhahran 31261, Saudi Arabia
kabbaj@kfupm.edu.sa

(Joint work with A. Mimouni)

dedicated to Alain Bouvier

The t-class semigroup of an integral domain R, denoted $S_t(R)$, is the semigroup of fractional t-ideals modulo its subsemigroup of nonzero principal ideals with the operation induced by ideal t-multiplication. We recently proved that if R is a Krull-type domain in the sense of Griffin [1], then $S_t(R)$ is a Clifford semigroup. In this work, we extend Bazzoni and Salce's study of groups in the class semigroup of a valuation domain or of a Prüfer domain of finite character to a larger class of integral domains. Precisely, we describe the idempotents of $S_t(R)$ and the structure of their associated groups when R is a Krull-type domain. Indeed, we prove that there are two types of idempotents in $S_t(R)$: those represented by fractional overrings of R and those represented by finite intersections of t-maximal ideals of fractional overrings of R.

References

[1] M. Griffin, Rings of Krull type, J. Reine Angew. Math. **229** (1968) 1–27.

Rings with elementary Artinian modules

A. Kaidi
Department of Algebra and Mathematical Analysis
University of Almeria
Almeria 04120, Spain
elamin@ual.es

(Joint work with A. Elmejdani)

Over a Dedekind domain D, every Artinian D-module M is a direct sum of a module of finite length and a finite number of injective hulls of simple modules, $M = F \oplus (\bigoplus_{i=1}^{n} E(S_i))$. In this talk, we provide more classes of rings over which Artinian modules have the same decomposition, and we give characterizations of those rings. We also study the rings over which the injective hull of any simple module is Artinian almost finitely generated (strongly co-Noetherian rings) and their connection with the structure of Artinian modules.

References

[1] J. P. Jans, On co-Noetherian rings, J. London Math. Soc. **2-1** (1969) 588–590.

[2] P. Vamos, The dual of the notion of "finitely generated," J. London Math. Soc. **43** (1968) 643–646.

[3] W. D. Weakley, Modules whose proper submodules are finitely generated, J. Algebra **84** (1983) 189–219.

Over a rank-one non-discrete valuation domain V the ring $V[[X]]_{V-(0)}$ is never a Prüfer domain

B. G. Kang
Department of Mathematics
Pohang University of Science and Technology
Pohang 790-784, Korea
`bgkang@postech.ac.kr`

(Joint work with M. H. Park)

Let V be a rank-one non-discrete valuation domain. While the ring of entire functions is a Bezout ring, we show that $V[[X]]_{V-(0)}$ is not even a Prüfer domain. This is an answer to Eakin and Sathaye's question of 1982.

Artinian subrings of a product of zero-dimensional rings

Driss Karim
Department of Mathematics
Faculty of Nador
University Mohamed the First
Nador 62700, Morocco
ikarim@ucam.ac.ma

Given a family of commutative rings $\{R_\alpha\}_{\alpha\in A}$, we investigate the family of Artinian subrings of $\prod_{\alpha\in A} R_\alpha$. Let R be a subring of a ring T, we say that (R,T) is a zero-dimensional pair if each intermediate ring between R and T is zero-dimensional. We also examine the question of when a pair $(\prod_{\alpha\in A} R_\alpha, \prod_{\alpha\in A} T_\alpha)$ is zero-dimensional, where $\{(R_\alpha, T_\alpha)\}_{\alpha\in A}$ is a family of zero-dimensional pairs.

Bouvier's contribution to dimension theory

M. Khalis
Department of Mathematics
FST Settat
Settat, Morocco
mohammed_khalis@hotmail.com

dedicated to Alain Bouvier

This lecture sheds light on Alain Bouvier's recent contribution in spectra and dimension theory of polynomial rings. The first part deals with different properties of the Krull and valuative dimensions and the related topic of Jaffard domain. The second part studies the universal catenarity of integral domains; particularly, LFD Prüfer domains, domains of global dimension 2, and GD-domains. Correlations to the altitude formula and Kaplansky's strong S-properties are also emphasized.

Trace properties in rings with zero divisors

Thomas G. Lucas
Department of Mathematics and Statistics
University of North Carolina Charlotte
Charlotte, NC 28223, USA
`tglucas@uncc.edu`

(Joint work with Dawn McNair)

An ideal is semiregular if it contains a finitely generated dense ideal, and it is regular if it contains an element that is not a zero divisor. The trace of a dense ideal I of a ring R is the (trace) ideal $I(R:I)$ where $(R:I) = \{t \in Q(R) \mid tI \subseteq R\}$ ($Q(R)$ denotes the complete ring of quotients of R). Extending definitions for trace properties of integral domains (see [1] and [2]), we say that R is an (Q_0-)RTP ring if the trace of each noninvertible (semi)regular ideal is a radical ideal, and R is an (Q_0-)LTP ring if $IR_P = PR_P$ for each (semi)regular trace ideal I and each prime P minimal over I. If R is an RTP ring (Q_0-RTP ring) and I is a (semi)regular ideal of R, then R/I is a Q_0-RTP ring. One consequence of this is that the homomorphic image of an RTP domain is always a Q_0-RTP ring. Similar statements hold for LTP rings, Q_0-LTP rings and LTP domains.

References

[1] W. Heinzer and I. Papick, The radical trace property, J. Algebra 112 (1988), 110-121.

[2] S. Kabbaj, T. Lucas and A. Mimouni, Trace Properties and Integral Domains, in Advances in Commutative Ring Theory, 421–436, Lecture Notes in Pure and Applied Mathematics, 205, Marcel Dekker, New York, 1999.

[3] D. McNair, Duals of ideals and trace properties in rings with zero divisors, Thesis, University of North Carolina Charlotte, 2008.

Sufficient condition to resolve Costa's first conjecture

N. Mahdou
Department of Mathematics
Faculty of Science and Technology
University S. M. Ben Abdellah
Fez, Morocco
mahdou@hotmail.com

(Joint work with S. Kabbaj)

dedicated to Alain Bouvier

The goal of this paper is to give a sufficient condition to resolve Costa's conjecture for each positive integers n and d with $n \geq 4$. Precisely, we show that if there exists a local ring (A,M) such that $\lambda_A(M) = n$ and an $(n+2)$-presented A-submodule of M^m, where m is a positive integer (In particular, if M contains a regular element), then one can construct an example of $(n+4,d)$-ring which is neither $(n+3,d)$-ring nor an $(n+4,d-1)$-ring. Finally, we construct a local ring (B,M) such that $\lambda_B(M) = 0$ (resp., $\lambda_B(M) = 1$) and so we exhibit for each positive integer d, an example of $(4,d)$-ring (resp., $(5,d)$-ring) which is neither a $(4,d-1)$-ring (resp., neither a $(5,d-1)$-ring) nor a $(2,d')$-ring (resp., nor a $(3,d')$-ring) for each positive integer d'.

References

[1] D. Costa, Parameterizing families of non-Noetherian rings, Comm. Algebra 22 (10) (1994) 3997–4011.

[2] S. Kabbaj, N. Mahdou, Trivial extensions of local rings and a conjecture of Costa, Lect. Notes Pure Appl. Math. Dekker 231 (2003) 301–312.

[3] N. Mahdou, On Costa's conjecture, Comm. Algebra 29 (7) (2001) 2775–2785.

t-Class semigroups of Noetherian domains

A. Mimouni
Department of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
Dhahran 31261, Saudi Arabia
amimouni@kfupm.edu.sa

(Joint work with S. Kabbaj)

dedicated to Alain Bouvier

Let R be an integral domain. The class semigroup of R, denoted $\mathscr{S}(R)$, is the semigroup of nonzero fractional ideals modulo its subsemigroup of nonzero principal ideals [1, 2]. We define the t-class semigroup of R, denoted $\mathscr{S}_t(R)$, as the t-analogue of $\mathscr{S}(R)$, that is, the semigroup of fractional t-ideals modulo its subsemigroup of nonzero principal ideals with the operation induced by t-multiplication. In this talk we will investigate ring-theoretic properties of a Noetherian domain that reflect reciprocally in the Clifford or Boolean property of its t-class semigroup.

References

[1] S. Bazzoni and L. Salce, Groups in the class semigroups of valuation domains, Israel J. Math. 95 (1996) 135–155.

[2] P. Zanardo and U. Zannier, The class semigroup of orders in number fields, Math. Proc. Cambridge Phil. Soc. 115 (1994) 379–391.

Infinite Hilbert 2-class field tower of quadratic number fields

A. Mouhib
Faculty of Taza
B/P 1223, Taza Gare
Taza, Morocco
mouhibali@yahoo.fr

Let k be a number field and C_k the ideal class group of k. Let k^1 be the Hilbert 2-class field of k (i.e., the maximal Abelian unramified 2-extension of k), and for $n \geq 2$, let k^n be the Hilbert 2-class field of k^{n-1}. Then

$$k \subset k^1 \subset k^2 \subset ... \subset k^n \subset ...$$

is the Hilbert 2-class field tower of k. We say that the tower is finite if $k^n = k^{n+1}$ for some n, and infinite otherwise. We define the 2-rank of C_k as the dimension of the elementary Abelian 2-group C_k/C_k^2 viewed as a vector space over $\mathbb{F}_2$:

$$rank_2(C_k) = \dim_{\mathbb{F}_2}(C_k/C_k^2),$$

where $\mathbb{F}_2$ is the finite field with two elements. Suppose now that k is an imaginary quadratic number field. In [Tours de corps de classes et estimations de discriminants, Invent. Math. 44 (1978), 65-73], J. Martinet conjectured that if $rank_2(C_k) = 4$, then k has infinite Hilbert 2-class field tower. In this article, we give a positive answer to this conjecture in the case where the discriminant of k is divisible by at most one negative prime discriminant.

References

[1] A. Mouhib et A. Movahhedi, Sur le 2-groupe de classes des corps multiquadratiques réels. Journal de théorie des nombres de Bordeaux **17** (2005) 619–641.

Cofinitenes and associated primes of local cohomology modules over Noetherian rings

R. Naghipour
Department of Mathematics
University of Tabriz
Tabriz 51666-16471, Iran
`naghipour@ipm.ir`

(Joint work with K. Bahmanpour)

Let R be a (not necessarily local) Noetherian ring and let M be a non-zero finitely generated R-module. Let I be an ideal of R and t a non-negative integer, such that $\dim H_I^i(M) \leq 1$ for all $i < t$. It is shown that the R-modules $H_I^0(M), H_I^1(M), ..., H_I^{t-1}(M)$ are I-cofinite and the R-module $\mathrm{Hom}_R(R/I, H_I^t(M))$ is finitely generated. This immediately implies that if $\dim R/I = 1$, then $H_I^i(M)$ is I-cofinite for all $i \geq 0$. This is a generalization of the main results of Delfino-Marley [1] and Yoshida [2]. Also, we prove that if R is local and $\dim H_I^i(M) \leq 2$ for all $i < t$, then the R-modules

$$\mathrm{Ext}_R^j(R/I, H_I^i(M)) \text{ and } \mathrm{Hom}_R(R/I, H_I^t(M))$$

are weakly Laskerian for all $i < t$ and all $j \geq 0$. Consequently, it follows that the set of associated primes of $H_I^i(M)$ is finite for all $i \geq 0$, whenever $\dim R/I \leq 2$.

References

[1] D. Delfino and T. Marley, Cofinite modules and local cohomology, J. Pure Appl. Algebra **121** (1997) 45–52.

[2] K.I. Yoshida, Cofiniteness of local cohomology modules for ideals of dimension one, Nagoya Math. J. **147** (1997) 179–191.

Indecomposability of polynomials via Jacobian matrix

S. Najib
Max-Planck Institute-Bonn
Bonn 53111, Germany
najib@mpim-bonn.mpg.de ; najibm@voila.fr

(Joint work with G. Chèze)

Uni-multivariate decomposition of polynomials is a special case of absolute factorization. Recently, thanks to the Ruppert's matrix some effective results about absolute factorization have been improved. We show that with a Jacobian matrix we can get sharper bounds for the special case of uni-multivariate decomposition.

References

[1] I. V. Arzhantsev and A.P. Petravchuk, Closed and irreducible polynomials in several variables, arXiv:math/0608157v2, 20 May 2007.

[2] G. Chèze, Des méthodes symboliques-numériques et exactes pour la factorisation absolue des polynômes en deux variables, Thèse de Doctorat, Univ. Nice-Sophia Antipolis, 2004.

[3] S. Gao, Factoring multivariate polynomials via partial differential equations, Math. Comp. **72** (2003) 801–822.

[4] S. Najib, Une généralisation de l'inégalité de Stein-Lorenzini, J. Algebra **292** (2005) 566–573.

[5] W. M. Ruppert, Reduzibilität ebener Kurven, J. Reine Angew. Math. **369** (1986) 167–191.

Bouvier's contribution in multiplicative ideal theory

D. Nour Elabidine
Department of Mathematics
FST Mohammedia
Mohammedia, Morocco
nourelabidine@hotmail.com

dedicated to Alain Bouvier

In this lecture we are going to highlight the work of Alain Bouvier in the area of non-Noetherian commutative ring theory which is now receiving considerable attention by several active groups all over the world. Bouvier's work made a valuable addition to the literature and helped stimulate a lot of research in the theory of class groups and divisibility.

Derivations, formal fibers and bad Noetherian rings

B. Olberding
Department of Mathematical Sciences
New Mexico State University
Las Cruces, NM 88003-8001
olberdin@nmsu.edu

Let $A \subseteq S$ be an extension of domains having quotient fields Q and F, respectively. In this talk we consider the somewhat special situation where $F = S + Q$ and $A = S \cap Q$. If such a relation is satisfied, then $A \subseteq S$ is necessarily faithfully flat (in fact, it is pure), and hence there is a strong relationship between A and S. An example of such a phenomenon is where K is a field, $S = K[[X]]$ and $A = K[X]_{(X)}$. Indeed, $K((X)) = K(X) + K[[X]]$, and $K[[X]] \cap K(X) = K[X]_{(X)}$. Thus, roughly speaking, for the extensions $A \subseteq S$ we are interested in, the elements of F can be expressed "meromorphically" via Q and S. For such an extension, we are specifically interested in the A-subalgebras R of S such that $QR = F$, and hence have quotient field F. Such rings R have many peculiar properties. For example, their completions (where the notion of completion is allowed to vary here over several possible choices of linear topologies) are isomorphic to an idealization of the completion of S, and hence ramify in a strong way. (And when R is Noetherian this is what makes them, in Nagata's terminology, "bad.") Moreover, such rings R are intimately connected to S-submodules of $\Omega_{F/Q}$, the module of Kähler differentials of the extension F/Q. Exploiting this fact allows one to construct a wide range of bad Noetherian, and Noetherian-like, rings. Valuations are specifically useful for this purpose, and make it possible to find bad Noetherian rings in "good" places; e.g., as overrings of affine domains.

Gorenstein global dimension of trivial ring extensions

K. Ouarghi
Department of Mathematics
Faculty of Science and Technology
University S. M. Ben Abdellah
Fez, Morocco
ouarghi.khalid@hotmail.fr

(Joint work with N. Mahdou)

dedicated to Alain Bouvier

It is known that the global dimension of trivial ring extensions is often infinite. Recently, Bennis and Mahdou [2] introduced and studied the Gorenstein global dimension of rings. In this paper, we compute the Gorenstein global dimension for some trivial ring extensions and show that it remains also infinite. This study provides us with a counter-example showing that the transfer of the notion of Gorenstein projective module does not hold in pullback constructions in general.

References

[1] D. Bennis and N. Mahdou, Strongly Gorenstein projective, injective, and flat modules, J. Pure Appl. Algebra **210** (2007) 437–445.

[2] D. Bennis and N. Mahdou, Global Gorenstein dimensions, submitted for publication. (arXiv: math.AC/0611358)

[3] L. W. Christensen, Gorenstein dimensions, Lecture Notes in Math., 1747, Springer, Berlin, 2000.

[4] E. E. Enochs and O. M. G. Jenda, Relative homological algebra, Expositions in Mathematics, vol. 30, Walter de Gruyter & Co., Berlin, 2000.

[5] H. Holm, Gorenstein homological dimensions, J. Pure Appl. Algebra **189** (2004) 167–193.

Straight rings

G. Picavet
Laboratoire de Mathématique
Université Blaise Pascal (Clermont II)
63177 Aubière Cedex, France
Gabriel.Picavet@math.univ-bpclermont.fr

(Joint work with D. Dobbs)

We define a new class of rings (and domains), related to divived rings and going-down rings. A ring morphism $A \to B$ is called prime if B/PB is torsion-free over A/P for each prime ideal P of A. Then a ring extension $R \subseteq S$ is called straight if each of its subextensions is prime. A ring A is dubbed extensionally straight if A is straight in $\mathrm{Tot}(A)$ and straight if A/P is extensionally straight for each prime ideal P of A. Actually, a straight domain is nothing but an extensionally straight domain. The following implications hold: Locally divided $\Rightarrow$ straight $\Rightarrow$ going-down. None of these implications can be reversed in general. However, they can be reversed for wide classes of rings, such as the class of seminormal rings. The stability of the class of (extensionally) straight rings with respect to usual algebraic constructions is discussed.

One part of the above material was already given at a talk during the Algebra Conference, Venezia 2002. Since then, many progresses have been accomplished. A first paper is to appear soon in Comm. Algebra. A second paper need only to be couched down.

Characterization of rings with only finitely many subrings

M. Picavet
Laboratoire de Mathematiques
Université Blaise Pascal
63177 Aubière Cedex France
`Martine.Picavet@math.univ-bpclermont.fr`

(Joint work with D.E. Dobbs and G. Picavet)

We give a complete characterization of commutative rings which have only finitely many subrings. The two main results are the following :

A ring R has only finitely many subrings if and only if R is of the form $F[t_1,\ldots,t_n]$, where $F[t_i]$ has only finitely many subrings for each $i = 1,\ldots,n$ and where F is the prime ring of R.

Now, let R be a singly generated ring over its prime ring F. Then R has only finitely many subrings if and only if (exactly) one of the following four conditions holds :

(1) R is a finite ring;

(2) $R \simeq \mathbb{Z}[1/n]$ for some positive integer n;

(3) R is a module-finite ring extension of $\mathbb{Z}$, which is not a domain, and the conductor $(\mathbb{Z} : R) = \{r \in R \mid rR \subseteq \mathbb{Z}\}$ is nonzero;

(4) R is of the form $\mathbb{Z}[t]$, is not integral over $\mathbb{Z}$, is not an integral domain, there exist integers $a,b \geq 2$ such that $at = b$, and the greatest common divisor $d = (a,b)$ of the minimal such a and the corresponding b satisfies the following conditions : $d > 1$, αt is integral over $\mathbb{Z}$, where $a = \alpha d$ and there does not exist a prime number p such that $\ker(\varphi) \subseteq p\mathbb{Z}[X]$, where φ is the ring homomorphism $\varphi : \mathbb{Z}[X] \to R$ sending X to t.

We also get the following characterization for direct products of rings :

A nontrivial ring direct product of ring $\prod_{i\in I} R_i$ has only finitely many subrings if and only if I is finite, each R_i has only finitely many subrings, and there is at most one $i \in I$ such that R_i has characteristic 0.

A generalization of stable domains

Giampaolo Picozza
Faculté des Sciences et Techniques
Université Paul Cézanne
13397, Marseille Cedex 20. France
giampaolo.picozza@univ-cezanne.fr

(Joint work with Francesca Tartarone)

In this work in progress with Francesca Tartarone, we introduce, as a natural generalization of stable domains (domains in which each ideal is projective in its endomorphism ring), the domains in which each ideal is flat in its endomorphism ring, which we have called "flat-stable domains". We give some general result on flat ideals, some example of flat-stable domain and show that this class is strictly included between the class of stable domains and the class of finitely stable domains (the domains in which each finitely generated ideal is projective in its endomorphism ring). Moreover, we study how this property transfers to localizations and to general overrings.

Transfinite self-idealizations and commutative rings of triangular matrices

L. Salce
Dipartimento di Matematica Pura e Applicata
Univerità di Padova
Via Trieste, 63, 35121 Padova, Italy
salce@math.unipd.it

The self-idealization of a commutative ring is iterated countably many times, producing an inverse-direct system of rings. We investigate the rings which are the inverse limit and the direct limit of this system. A slight modification of this construction, iterated uncountably many times, gives rise to modules resembling the uncountably generated uniserial modules over valuation domains and their clones, namely, the non-standard uniserial modules.

Gorenstein Artin algebras with Hilbert function $(1,4,7,...,7,4,1)$

Hema Srinivasan
Department of Mathematics
University of Missouri
Columbia, MO 65211, USA
srinivasanh@missouri.edu

Let R be an Artinian Gorenstein Ring of embedding dimension four. We will give a general structure for a class of these ideals with a Hilbert function $(1,4,7,...,7,4,1)$. If after a change of variables, the three quadratic generators form an ideal of height one, we will give the structure of the free resolution for these ideals. This extends the results of Iarrobino and Srinivasan. Some of these are joint work with my student Sabine El-Khoury.

Let $R = k[x,y,z,w]$ be the polynomial ring and I be a height four homogeneous ideal with $H(I) = (1,4,7,...,7,4,1)$. Suppose I_2 has height 1. If $J = I \cap k[x,y,z]$, then J is either Gorenstein or of type two. If J is Gorenstein, then we have the structure from the theorem of Iarrobino and Srinivasan. If J is of type two, then we get the structure of these mysterious Gorenstein ideals I built from the resolution of R/J.

Integral closure – a new algorithm

Irena Swanson
Department of Mathematics
Reed College
Portland, OR 97202, USA
iswanson@reed.edu

(Joint work with Anurag Singh)

I will present the history of the computation of integral closures, starting with Dedekind's determination of the integral closures of cyclic extensions of the ring of integers, and ending with our recent joint work.

I will concentrate mostly on the algorithmic aspects of the computation. The first algorithmic consideration is due to Stolzenberg from 1975, and was improved by Seidenberg. A more effective method for computing the integral closure of affine domains is due to Grauert, Remmert, and de Jong, and further modifications and refinements are due to Vasconcelos. These algorithms successively approximate the integral closure from below, namely by building successively strictly larger rings between the original ring and its integral closure. Based on a specialized 2003 algorithm of Leonard–Pellikaan [2] we prove a more general version of the construction of the integral closure that starts instead with a finitely generated module over the ring that contains the integral closure, and the successive steps produce strictly smaller submodules that contain the integral closure. We also prove a new algorithm for computing the integral closure of ideals. We implemented both algorithms in Macaulay2 [1].

References

[1] D. Grayson and M. Stillman, Macaulay2. 1996. A system for computation in algebraic geometry and commutative algebra, available via anonymous `ftp` from `math.uiuc.edu`.

[2] D. A. Leonard and R. Pellikaan, Integral closures and weight functions over finite fields, Finite Fields Appl. **9** (2003) 479–504.

Integrally closed overrings of $\mathbb{Z}[X]$

F. Tartarone
Department of Mathematics
Università degli Studi "Roma Tre"
Largo San Leonardo Murialdo 1, 00146 Roma, Italy
tfrance@mat.uniroma3.it

(Joint work with A. Loper)

It is well-known that the ring $\text{Int}(\mathbb{Z}) = \{f \in \mathbb{Q}[X] \mid f(\mathbb{Z}) \subseteq \mathbb{Z}\}$ is a Prüfer domain and the set of its valuation overrings is completely known (in particular, the set of the valuation overrings in which a fixed prime number p is not invertible is in bijection with the p-adic completion of $\mathbb{Z}$, $\widehat{\mathbb{Z}_p}$). We generalize this fact giving a characterization of the Prüfer overrings of $\mathbb{Z}[X]$, by describing the set of their valuation overrings. In particular we fix a prime number $p \in \mathbb{Z}$ and consider the set T_p of the valuation overrings of $\mathbb{Z}_p[X]$. On T_p we put an order described by S. MacLane in [1] and study properties of some sub-collections of T_p. In particular, if R is an integrally closed domain, with $\mathbb{Z}_p[X] \subseteq R \subseteq \mathbb{Q}[X]$, we denote by $T_p(R)$ the set of the valuation overrings of R (then $T_p(R) \subseteq T_p$). We find results which allows to use some of these order-theoretic information on $T_p(R)$ to study the multiplicative ideal structure of R.

References

[1] S. MacLane, A construction for absolute values in polynomial rings, Trans. Amer. Math. Soc. **40** no. 3 (1936) 363–395.

Monoids of modules

Roger Wiegand
Department of Mathematics
University of Nebraska
Lincoln, NE 68588-0130, USA
rwiegand@math.unl.edu
(Joint work with Sylvia Wiegand)

I will survey some developments in the study of direct-sum decompositions of modules. The idea is to examine direct-sum behavior by looking at monoids of isomorphism classes of modules. To be precise, let R be a ring, and let $\mathscr{C}$ be a class of modules closed under isomorphism, finite direct sums, and direct summands. Assume there is a set $\mathrm{V}(\mathscr{C}) \subseteq \mathscr{C}$ of representatives; thus each element $M \in \mathscr{C}$ is isomorphic to exactly one element $[M] \in \mathrm{V}(\mathscr{C})$. We make $\mathrm{V}(\mathscr{C})$ into an additive monoid in the obvious way: $[M]+[N] = [M \oplus N]$. This monoid encodes information about the way modules in $\mathscr{C}$ decompose as direct sums of other modules in $\mathscr{C}$.

For example, given a module M, we let $+(M)$ be the class of modules that are isomorphic to direct summands of direct sums of finitely many copies of M. If R is a (commutative Noetherian) local ring and M is a finitely generated R-module, then $\mathrm{V}(+(M))$ is a finitely generated Krull monoid. Conversely, there is a "realization theorem" [1]: given any finitely generated Krull monoid H, there is local ring R with a finitely generated R-module M such that $\mathrm{V}((+M)) \cong H$. Using the realization theorem, one can find an indecomposable finitely generated module M over a local ring such that $M, M^{(2)}, \ldots, M^{(100)}$ have only the obvious direct-sum decompositions, whereas $M^{(101)} \cong V \oplus W$, where V and W are non-isomorphic indecomposables. On the other hand, using the fact that $\mathrm{V}(+(M))$ is a Krull monoid, one can show that if M and N are finitely generated indecomposable modules over a local ring and $M^{(p)} \cong N^{(q)}$ for positive integers p and q, then $p = q$ and $M \cong N$. I will describe some more subtle restrictions on direct-sum relations that emerge from this point of view.

References

[1] R. Wiegand, Direct-sum decompositions over local rings, J. Algebra **240** (2001), 83–97.

[2] R. Wiegand and S. Wiegand, Semigroups of modules: a survey, to appear in Proceedings of the International Conference on Rings and Things, Zanesville, Ohio, 2007.

Prime ideals in power series rings and relations among spectra

S. Wiegand
Department of Mathematics
University of Nebraska Lincoln
Lincoln NE USA
swiegand@math.unl.edu

(Joint work with W. Heinzer and C. Rotthaus)

We consider prime ideals in mixed polynomial power series rings of the form k[[x]][y] and k[y][[x]] and their relations when one such ring is contained in another. We give some pictures of the prime spectra. Also we give some results concerning a question of Hochster and Yao: "What can you say about a complete local domain S that is an extension of another complete local domain R such that the maximal ideal of S intersects R in its maximal ideal and the 0 ideal of S is the only ideal of S that intersects R in 0?"

Local cohomology for modules finite over a local homomorphism

Siamak Yassemi
Department of Mathematics, University of Tehran
and Institute for Theoretical Physics and Mathematics (IPM)
Tehran, Iran
siamak.yassemi@gmail.com

(Joint work with M. Tousi)

Let $(R,\mathfrak{m})$ be a Noetherian local ring and let $\mathfrak{a}$ be a proper ideal of R. Let $\mathrm{H}^i_{\mathfrak{a}}(-)$ denote the i-th local cohomology module of $-$ with support in $\mathrm{V}(\mathfrak{a})$. It is of great importance in algebraic geometry and commutative algebra to have (non-)vanishing results for the local cohomology modules $\mathrm{H}^i_{\mathfrak{a}}(M)$, $i \in \mathbb{N}$, for an R-module M. Our approach in this talk is to study (non-)vanishing results for the local cohomology for modules finite over a local homomorphism. Recall that an R-module M is called finite over a local homomorphism $\varphi\colon (R,\mathfrak{m}) \to (S,\mathfrak{n})$, if M is a finitely generated S-module. One would expect of modules finite over a local homomorphism to behave, in a way, very much like finitely generated ones. But this is not true in general.

Betti numbers of transversal monomial ideals

Rahim Zaare-Nahandi
Department of Mathematics, Statistics and Computer Science
University of Tehran
Tehran, Iran
rahimzn@khayam.ut.ac.ir

Let $R = k[y_{ij_i} : 1 \leq i \leq n, 1 \leq j_i \leq b_i]$ be the polynomial ring in $m = b_1 + \cdots + b_n$ indeterminates over a field k. Let $\mathbf{D} = \mathbf{D}(b_1, b_2, \ldots, b_n)$ be the matrix

$$\begin{bmatrix} y_{11} & \cdots & y_{1b_1} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & y_{21} & \cdots & y_{2b_2} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & y_{n1} & \cdots & y_{nb_n} \end{bmatrix}.$$

Let $I_t(\mathbf{D})$ be the *transversal monomial ideal* in R generated by t-minors of $\mathbf{D}$. The ideal $I_t(\mathbf{D})$ is a square-free monomial ideal and the minimal free resolution of $R/I_t(\mathbf{D})$ is known. We outline a modification of this resolution and compute the Betti numbers of $I_t(\mathbf{D})$.

Let $S = k[x_{ij} : 1 \leq i \leq n, 1 \leq j \leq b]$ be the polynomial ring in nb indeterminates over k and let $\mathbf{P} = [C_1 C_2 \cdots C_b]$ be a *generic pluri-circulant matrix* where C_j is the circulant matrix with the first row $\begin{pmatrix} x_{1j} & x_{2j} & . & . & . & x_{nj} \end{pmatrix}$. If k possesses the nth roots of unity and $\mathrm{char}(k) \nmid n$ then $S/I_t(\mathbf{P}) \cong R/I_t(\mathbf{D})$ with $b_1 = b_2 = \cdots = b_n = b$. It is known that, for a suitable monomial order on S, certain set of t-minors of the first t rows of $\mathbf{P}$ forms a Gröbner basis for $I_t(\mathbf{P})$. We show that for $b = 2$, $\mathrm{in}(I_t(\mathbf{P}))$, the initial ideal of $I_t(\mathbf{P})$, is a *stable monomial ideal*. Using the Eliahou-Kervaire resolution for stable monomial ideals, we show that all Betti numbers of $\mathrm{in}(I_t(\mathbf{P}))$ and $I_t(\mathbf{D})$ are equal.

Betti numbers of the associated graded rings of certain monomial curves

Rashid Zaare Nahandi
Department of Mathematics
Institute for Advanced Studies in Basic Sciences (IASBS)
Zanjan 45195-Iran
rashidzn@iasbs.ac.ir

(Joint work with F. Rahmati and L. Sharifan)

Let $A = K[t^{a_1}, \ldots, t_n^a]$ be the coordinate ring of a monomial curve **C** corresponding to the numerical semigroup S minimally generated by a sequence $a_1, ..., a_n$. Let $\mathrm{gr}_{\mathfrak{m}}(A) = \bigoplus_{i \geq 0}(\mathfrak{m}^i/\mathfrak{m}^{i+1})$ be the associated graded ring of A with respect to the maximal ideal $\mathfrak{m} = \langle t_{a_1}, \ldots, t^{a_n} \rangle$.

In this talk we present the Betti numbers of $\mathrm{gr}_{\mathfrak{m}}(A)$, in the case that it is Cohen-Macaulay and

$$\mu = e + d - 2$$

where d is dimension, e is multiplicity and μ is the embedding dimension of the ring A.

References

[1] Bruns, W., Herzog, J. *Cohen-Macaulay Rings*, Cambridge University Press, Cambridge, 1993.

[2] S. Eliahou and M. Kervaire, Minimal resolution of some monomial ideals, *J. Algebra* 129 (1990) no. 1, 1–25.

[3] J. D. Sally, On the associated graded rings of a local Cohen-Macauley rings, *J. Math. Kyoto Univ.* 17 (1977) 19–21.

[4] J. D. Sally, Cohen-Macauley local rings of embedding dimension $e + d - 2$, *J. Algebra* 83 (1983) 393–408.

Algebraic entropy of endomorphisms over local one-dimensional domains

Paolo Zanardo
Dipartimento di Matematica
Università di Padova
Via Trieste 63, 35121 Padova, Italy
pzanardo@math.unipd.it

Let R be a local one-dimensional integral domain, with maximal ideal $\mathfrak{M}$ and field of fractions Q. We consider the algebraic entropy ent_g, defined using the invariant gen, where, for M a finitely generated R-module, $\mathrm{gen}(M)$ is its minimal number of generators. Our purpose is to relate some natural properties of R with the algebraic entropies $\mathrm{ent}_g(\phi)$ of the elements $\phi \in Q$, regarded as endomorphisms in $\mathrm{End}_R(Q)$. Specifically, let R be dominated by an Archimedean valuation domain V, with maximal ideal P. We examine the uniqueness of V, the transcendency of the residue fields extension V/P over $R/\mathfrak{M}$, and the condition for R to be a pseudo-valuation domain. We get mutual information between these properties and the behavior of ent_g, focusing on the conditions $\mathrm{ent}_g(\phi) = 0$ for every $\phi \in Q$, $\mathrm{ent}_g(\psi) = \infty$ for some $\psi \in Q$, and $\mathrm{ent}_g(\phi) < \infty$ for every $\phi \in Q$.