

On Q-normal Operators II
By

Adnan A. S. Jibril

Dept. of Math. & Statistics

King Fahd University of Petroleum & Minerals

Dhahran 31261, Saudi Arabia

Abstract

In this paper we continue studying Q-normal operators . We study the relation between the class (QN) of Q-normal operators and the class of quasinormal operator . We show that the class of quasinormal operators is strictly contained in (QN) .We give some conditions under which a Q-normal operator becomes quasinormal.

1. Let H be a complex Hilbert space and let $L(H)$ denotes the algebra of all bounded linear operators acting on H . For $T \in L(H)$ let T^* be its adjoint and let $T = A + iB$ be its cartesian decomposition. In [] we introduce the class (QN) of Q-normal operators :

$T \in L(H)$ is called Q-normal if $(T^*T)^2 = T^{*2}T^2$.We prove several basic properties and we study the relation between the class (QN) and the class (2N) of 2-normal operators introduced by the author in []. In this paper we study the relation between (QN) and the class of all quasinormal operators in $L(H)$. We show that the class of quasinormal operators is strictly contained in (QN).We give some conditions underwhich a Q-normal operator becomes quasinormal.

2. **Definiton 2.1.** $T \in L(H)$ is called quasinormal if $TT^*T = T^*T^2$.

Proposition 4.1. If $T \in L(H)$ is quasinormal then it is Q-normal.

Proof . Since T is quasinormal , $TT^*T = T^*T^2$. Now multiplying the last equation on

The left by T^* we get $(T^*T)^2 = T^{*2}T^2$.Thus T is Q-normal.

In the following we give an example of a Q-normal operator which is not quasinormal.

Example 2.1.

In the following we give a condition under which a Q-normal operator becomes quasinormal .

Proposition 4.2 . If $T \in L(H)$ is Q-normal such that $(T^*T)^3 = T^{*3}T^3$ then T is quasinormal .

Proof . $(T^*T - TT^*)^2 = (T^*T)^2 - T^*T^2T^* - TT^{*2}T + (TT^*)^2$ -----(i)

Multiplying (i) above on the left by T^* and on the right by T we get

$$T^*(T^*T - TT^*)^2T = T^*(T^*T)^2T - T^{*2}T^2T^*T - T^*TT^{*2}T^2 + T^*(TT^*)^2T$$
 -----(ii).

Since T is Q-normal , $(T^*T)^2 = T^{*2}T^2$ which implies when we substitute in (ii) that

$$T^*(T^*T - TT^*)^2T = T^{*3}T^3 - (T^*T)^3 - (T^*T)^3 + (T^*T)^3 .$$

$$= 0$$

Thus