King Fahd University of Petroleum and Minerals
Prep-Year Math Program

Exam II
Prep-Year Math I. Term (052)
April 27, 2006
Time Allowed: 75 Minutes

NAME:______________________________ **ID#** KEY **SEC#**________

IMPORTANT INSTRUCTIONS:

SHOW ALL YOUR WORK AND WRITE CLEAR STEPS

1) ALL TYPES OF CALCULATORS, PAGERS OR TELEPHONES ARE NOT ALLOWED DURING THE EXAMINATION.
2) WRITE YOUR NAME, ID NUMBER AND SECTION NUMBER.
3) USE ONLY PENCIL TO ANSWER THE QUESTIONS.
4) USE A GOOD ERASER, DON'T USE THE ERASER ATTACHED TO THE PENCIL.
5) CHECK THAT THE EXAM PAPER HAS 12 QUESTIONS.

1	2	3	4	5	6	7	8	9	10	11	12
3 pts	3 pts	2 pts	3 pts	3 pts	3 pts	3 pts	7 pts	4 pts	6 pts	5 pts	3 pts

TOTAL __________/45

1) **[3-points]** Find the center and the radius of the circle

$$4x^2 + 4y^2 - 24x + 16y - 12 = 0.$$

$\Rightarrow x^2 + y^2 - 6x + 4y - 3 = 0$

$\Rightarrow (x-3)^2 + (y+2)^2 = 3 + 9 + 4 = 16$ $\boxed{1}$

$\Rightarrow$ Center is at $(3, -2)$ $\boxed{1}$

radius $= \sqrt{16} = 4$ $\boxed{1}$

See example 7 p. 173
Problems 65 to 75 p 175

2) **[3-points]** A line segment has an endpoint at $(4, -6)$ and midpoint at $(-2, 11)$. Find the coordinates of the other endpoint.

Let (x, y) be the coordinates of the other endpoint

$\Rightarrow -2 = \frac{4+x}{2} \xRightarrow{\boxed{1}} -4 = 4 + x \Rightarrow x = -8$ $\boxed{0.5}$

and $11 = \frac{-6+y}{2} \xRightarrow{\boxed{1}} 22 = -6 + y \Rightarrow y = 28$

Thus the required point is $(-8, 28)$ $\boxed{0.5}$

See example 1 p. 174, Problems 19-24 p. 174 & 87-90 P. 176

3) **[2-points]** Find the distance between the points $(x, 4x)$ and $(-2x, 3x)$ where $x < 0$.
[Write your answer in simplest form]

The distance $= \sqrt{(x+2x)^2 + (4x-3x)^2}$

$= \sqrt{9x^2 + x^2} = \sqrt{10x^2}$ $\boxed{1}$

$= \sqrt{10}\,|x|$ $\boxed{0.5}$

$= -\sqrt{10}\,x$ $\boxed{0.5}$

It is problem 15 P. 174, see example 1 P. 160
and Problems 5 to 16 P. 174

4) **[3-points]** Determine whether the graph of the equation $|x-y|+|x+2y|=4$ is symmetric with respect to the:

A) $x-$axis $y \to -y \Rightarrow |x+y|+|x-2y|=4$

$\Rightarrow$ Not symmetric with respect to the x-axis $\boxed{1}$

B) $y-$axis $x \to -x \Rightarrow |-x-y|+|-x+2y|=4$

$\Rightarrow |x+y|+|x-2y|=4 \Rightarrow$

Not symmetric with respect to the y-axis $\boxed{1}$

C) Origin $x \to -x$ and $y \to -y \Rightarrow$

$|-x+y|+|-x-2y|=4 \Rightarrow |x-y|+|x+2y|=4$

$\Rightarrow$ Symmetric with respect to the origin. $\boxed{1}$

See examples 1, 2 p. 229–230 and problems 13 to 30 p. 238

5) **[3-points]** Find the value of the constant k for which the lines $kx+2y+5=0$ and $3x+(2k-1)y+7=0$ are **perpendicular**.

$kx+2y+5=0 \Rightarrow y=-\frac{k}{2}x-\frac{5}{2}$ $\boxed{0.5}$

$\Rightarrow$ its slope $m_1=-\frac{k}{2}$ $\boxed{0.5}$

and $3x+(2k-1)y+7=0 \Rightarrow y=-\frac{3}{2k-1}x-\frac{7}{2k-1}$ $\boxed{0.5}$

$\Rightarrow$ its slope $m_2=-\frac{3}{2k-1}$ $\boxed{0.5}$

$\Rightarrow m_1 m_2 = \left(-\frac{k}{2}\right)\left(\frac{-3}{2k-1}\right)=-1$ $\boxed{0.5}$

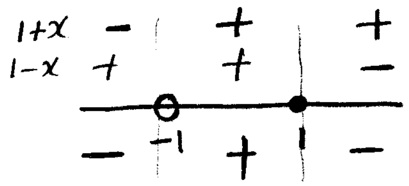
$\Rightarrow 3k=-4k+2 \Rightarrow 7k=2 \Rightarrow k=\frac{2}{7}$ $\boxed{0.5}$

See p. 206 and problems 25, 26 p. 208 and 76, 76 p. 211

6) [3-points] Determine, in interval notation, the domain of the function $f(x)=\sqrt{\dfrac{1-x}{1+x}}$ **[show your steps]**.

We must have $\dfrac{1-x}{1+x} \geqslant 0$ $\boxed{1}$

whose sign diagram is



$\boxed{1}$

The the domain $= (-1, 1]$ $\boxed{1}$

See example 4 p.182
and problems 27 to 38 p. 191

7) [3-points] A line L passes through the points $(\frac{1}{2},4)$ and $(\frac{7}{4},2)$. Find

A) The slope of L $= \dfrac{4-2}{\frac{1}{2}-\frac{7}{4}}$ $\boxed{0.5}$

$= \dfrac{2}{-\frac{5}{4}} = -\dfrac{8}{5}$ $\boxed{0.5}$

B) The equation of L in the form $y = mx + b$

See example 1 p. 199 and Problems 33 to 38 p. 208

$\Rightarrow$ $y = -\frac{8}{5}x + b$ $\boxed{0.5}$

and passing through $(\frac{1}{2}, 4)$ $\Rightarrow$

$4 = -\frac{4}{5} + b$ $\boxed{0.5}$

$\Rightarrow$ $b = 4 + \frac{4}{5} = \frac{24}{5}$ $\boxed{0.5}$

$\Rightarrow$ $y = -\frac{8}{5}x + \frac{24}{5}$ $\boxed{0.5}$

8) Given the function $f(x)=\begin{cases}\left[\frac{1}{2}x\right], & if \ -4\le x<2\\ 2x-4, & if \ \ 2\le x<3\\ 2, & if \ \ 3\le x\le 5\end{cases}$,

where $[y]$ is the greatest integer less than or equal to y.

A) **[4-points]** Find the value of each of the following:

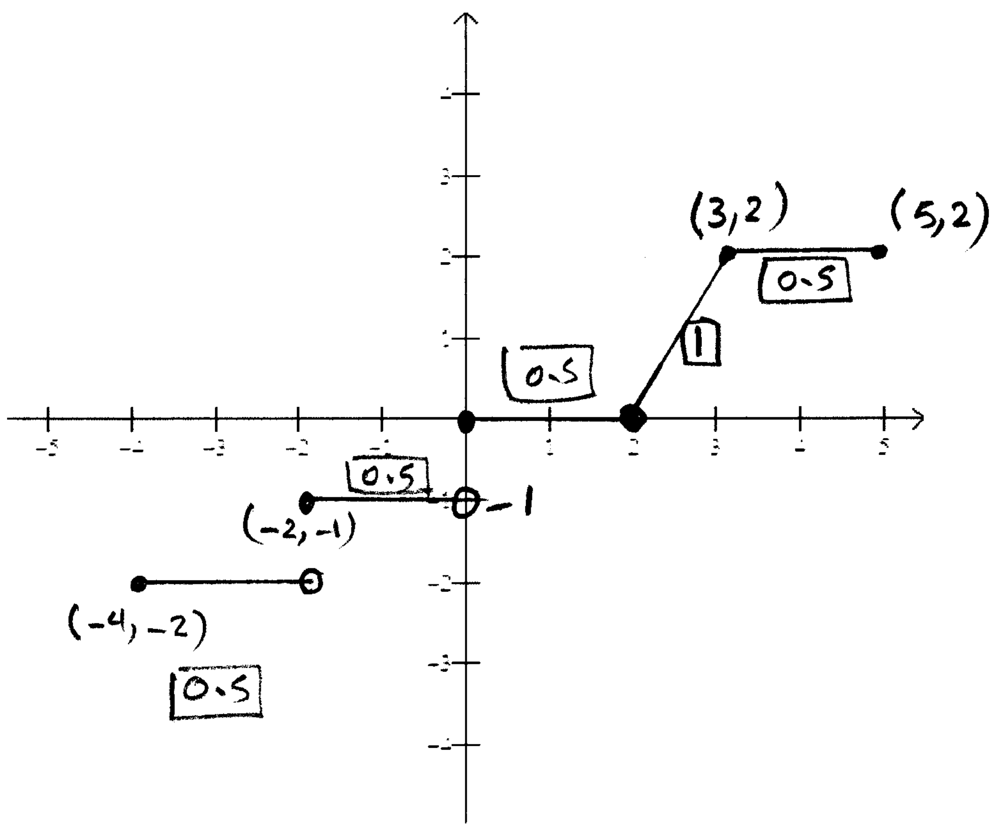
i) $f(\frac{-3}{2})=$ $\left[-\frac{3}{4}\right]=-1$ $\boxed{1}$

ii) $f(\sqrt{2})=$ $\left[\frac{\sqrt{2}}{2}\right]=0$ $\boxed{1}$

iii) $f(2.5)=$ $2(2.5)-4=5-4=1$ $\boxed{1}$

iv) $f(\pi)=$ 2 $\boxed{1}$

B) **[3-points]** Sketch the graph of f



9) **[4-points]** The graph of the equation $y = -3x^2 + 6x - 5$ is:

i) shifted left horizontally 1 unit, and shifted up vertically 2 units

then ii) reflected across the y-axis

then iii) compressed vertically by a factor of $\frac{1}{3}$ toward the x-axis.

Find the equation of the new graph [Show your steps].

i) $\Rightarrow y - 2 = -3(x+1)^2 + 6(x+1) - 5$

$\Rightarrow y = -3x^2 - 6x - 3 + 6x + 6 - 5 + 2$

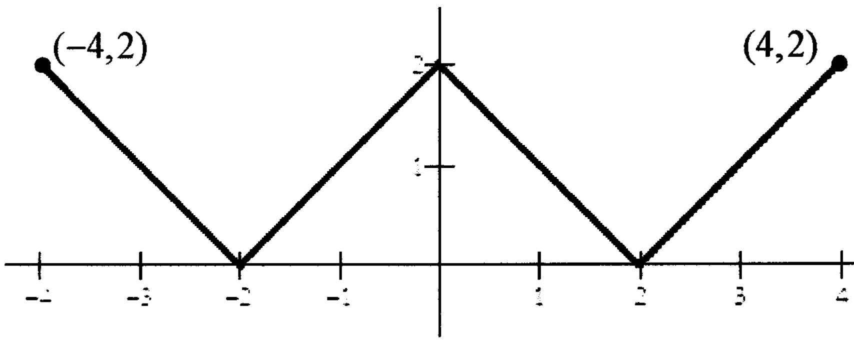
$\Rightarrow y = -3x^2$ $\boxed{1}$

ii) $\Rightarrow y = -3(-x)^2 = -3x^2$ $\boxed{1}$

iii) $\Rightarrow y = \frac{1}{3}(-3x^2) = -x^2$ $\boxed{1}$

See examples 4 p.233, 5 p.234, 6 p.235, 7 p.236

10) **[6-points]** Use the graph of the function f given in the figure to answer the following questions:



A) Find the domain of f

$\boxed{1}$ $= [-4, 4]$

B) Find the range of f

$\boxed{1}$ $= [0, 2]$

C) Find the interval(s) over which f is increasing

$\boxed{1}$ $[-2, 0] \cup [2, 4]$

D) Find the interval(s) over which f is decreasing

$\boxed{1}$ $[-4, -2] \cup [0, 2]$

E) Determine whether f is even, odd, or neither (why?) Even

$\boxed{1}$ Symmetric about the y-axis

F) Determine whether f is one-to-one (why?) Not one-to-one

$\boxed{1}$ By the horizontal line test.

See problems 51 to 62 p. 192

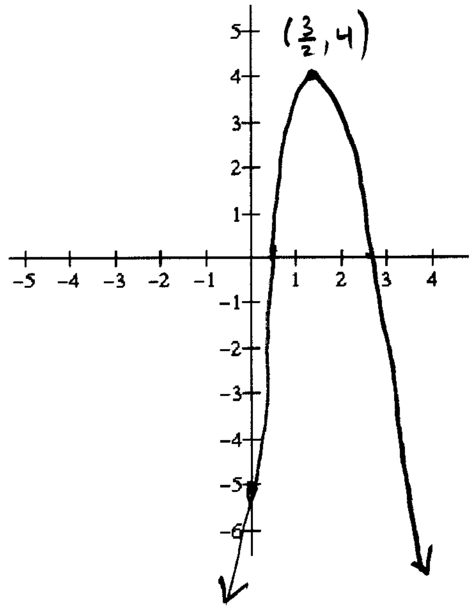
11) **[5-points]** Let f be the quadratic function $f(x) = -4x^2 + 12x - 5$.

i) Use the technique of **completing square** to find the standard form of f.

$$f(x) = -4(x^2 - 3x) - 5$$
$$= -4\left(x - \frac{3}{2}\right)^2 + 9 - 5 \quad \Rightarrow$$
$$f(x) = -4\left(x - \frac{3}{2}\right)^2 + 4 \qquad \boxed{1}$$

See examples 1 p. 215, 3 p. 217, 5 p. 319 & Problems 9 to 18 p. 222

ii) Sketch the graph of f, then determine the following:



A) The vertex of f.

$\left(\frac{3}{2}, 4\right)$ $\boxed{0.5}$

B) The maximum value of f (if exists).

$= 4$ $\boxed{0.5}$

C) The minimum value of f (if exists).

No minimum value $\boxed{0.5}$

D) The range of f, in interval notation.

$= (-\infty, 4]$

12) **[3-points]** Let f be an **odd function** such that $f(-2) = 3$. Find the coordinates of two points on the graph of the function $g(x) = 2f(3x)$. **[show your steps]**

f is odd and $f(-2) = 3 \Rightarrow f(2) = -3$ $\boxed{1}$

Now $g\left(\frac{2}{3}\right) = 2f(2) = -6$ $\boxed{0.5}$

and $g\left(-\frac{2}{3}\right) = 2f(-2) = 6$ $\boxed{0.5}$

$\Rightarrow$ Two points on g are $\left(\frac{2}{3}, -6\right)$ $\boxed{0.5}$ & $\left(-\frac{2}{3}, 6\right)$ $\boxed{0.5}$

See problems 61, 62, 65, 66 p. 239