Hwk 3

(1) Compute the pseudoinverse of $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$.

(2) Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $D \in \mathbb{R}^{m\times m}$ and suppose further that D is nonsingular. Prove or disprove that

$$\begin{bmatrix} A & AB \\ 0 & D \end{bmatrix}^{+} = \begin{bmatrix} A^{+} & -A^{+}ABD^{-1} \\ 0 & D^{-1} \end{bmatrix}$$

(3) Find the 1-, 2- and ∞- norms of $x = \begin{pmatrix} 2 \\ 1 \\ -4 \\ -2 \end{pmatrix}$ and $x = \begin{pmatrix} 1+i \\ 1-i \\ 1 \\ 4i \end{pmatrix}$.

(4) Show that $(\alpha_1 + \cdots + \alpha_n)^2 \le n(\alpha_1^2 + \cdots + \alpha_n^2)$ for $\alpha_i \in \mathbb{R}$.

(5) Find the Frobenius matrix norm for

$$C = \begin{pmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{pmatrix}$$

(6) Explain why $\|I\| = 1$ for every induced matrix norm? what is $\|I_{n\times n}\|_F$?

(7) For $n\times n$ matrices A and B, explain why

$$|\operatorname{trace}(B)|^2 \le n\,[\operatorname{trace}(B^* B)]$$

[8] Using the standard inner product, are the following pairs of vectors orthogonal.

$$x = \begin{pmatrix} i \\ 1+i \\ 2 \\ 1-i \end{pmatrix} \quad \text{and} \quad y = \begin{pmatrix} 0 \\ 1+i \\ -2 \\ 1-i \end{pmatrix} \quad \text{in } \mathbb{C}^4.$$

[9] Using the standard inner product, determine the Fourier expansion of x w.r.t. $\mathcal{B}$ where

$$x = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathcal{B} = \left\{ \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \right\}$$

[10] Use the Gram-Schmidt procedure to find an orthonormal basis for the fundamental subspaces of

$$A = \begin{pmatrix} 1 & -2 & 3 & -1 \\ 2 & -4 & 6 & -2 \\ 3 & -6 & 9 & -3 \end{pmatrix}.$$

[11] let $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & -3 \\ 0 & 1 & 1 \end{bmatrix}$.

Determine the rectangular QR-factorization of A.