Homework 2

1- Suppose the coefficient matrix of a non-singular system $Ax=b$ is updated to produce another non-singular system $(A+cd^T)z=b$, where $b, c, d \in \mathbb{R}^{n\times 1}$ and let y be the solution of $Ay=c$. Show that $z=x-yd^Tx/(1+d^Ty)$.

2. Let $[A\ B]$ be $n\times(k+m)$ matrix of full column rank and define

$$Z=[A\ B]^T[A\ B].$$

Let $M_A=I_n-A(A^TA)^{-1}A^T$, $M_B=I_n-B(B^TB)^{-1}B^T$,

and $E=B^TM_AB$, $F=A^TM_BA$.

Show that Z^{-1} can be expressed as:

$$Z^{-1}=\begin{pmatrix}(A^TA)^{-1}+(A^TA)^{-1}A^TBE^{-1}B^TA(A^TA)^{-1} & -(A^TA)^{-1}A^TBE^{-1}\\ -E^{-1}B^TA(A^TA)^{-1} & E^{-1}\end{pmatrix}$$

3 (a) If A and D are square, show that

$$\begin{pmatrix}A & B\\ 0 & D\end{pmatrix}^k=\begin{pmatrix}A^k & Q_k\\ 0 & D^k\end{pmatrix} \qquad k=1,2,\ldots.$$

where $Q_k=\sum_{j=1}^{k}A^{k-j}BD^{j-1}$

(b) What is Q_k if $D=I_n$ and I_m-A is nonsingular?

(c) Does the result in (a) also hold for ~~any~~ nonnegative integers?.

④ Let A and B be square matrices, Show that

$$\max\{\nu(A), \nu(B)\} \leq \nu(AB) \leq \nu(A) + \nu(B)$$

where $\nu(\star) = \dim N(\star)$ denotes the nullity.

⑤ Determine $\dim[N(A) \cap R(B)]$ for

$$A = \begin{bmatrix} -2 & 1 & 1 \\ -4 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 3 & 1 & -4 \\ -1 & -3 & 1 & 0 \\ 2 & 6 & 2 & -8 \end{bmatrix}$$