HWK 1:

① Determine which of the following subsets of $\mathbb{R}^n$ are in fact subspace of $\mathbb{R}^n$

(a) $\{x \mid x_1 x_2 = 0\}$ (b) $\{x \mid \sum_{j=1}^{n} x_j = 0\}$

(c) $\{x \mid \sum_{j=1}^{n} x_j = 1\}$.

② Determine which of the following subsets of $\mathbb{R}^{n\times n}$ are infact subspaces of $\mathbb{R}^{n\times n}$.

(a) The symmetric matrices (b) The diagonal matrices

(c) The singular matrices (d) all matrices s.th. $A^2 = A$.

③ Do the set $S' = \{(1\ 2\ 1), (2\ 0\ -1), (4\ 4\ 0)\}$ span $\mathbb{R}^3$?

④ Let V be a vector space and $\mathcal{M}, \mathcal{N} \subseteq V$. explain why $\operatorname{span}(\mathcal{M} \cup \mathcal{N}) = \operatorname{span}(\mathcal{M}) + \operatorname{span}(\mathcal{N})$.

⑤ Determine if the following sets of vectors are linearly independent or linearly dependent.

~~{(1 2 3)~~ (a) $\{(2\ 2\ 2\ 2), (2\ 2\ 0\ 2), (2\ 0\ 2\ 2)\}$

⑥ If T is triangular matrix in which each $t_{ii} \neq 0$ explain why the rows and columns of T must each be linearly independent sets.

(7) Determine whether or not the following set of matrices is linearly independent set:

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}.$$

(8) Which of the following sets of functions are linearly indep.?

(a) $\{x\sin x, \cos x, \sin x\}$.

(b) $\{e^x, xe^x, x^2e^x\}$

(c) $\{\sin^2 x, \cos^2 x, \cos 2x\}$.

(9) Let V = finite-dimensional vector space with $\dim(V) = n$.

(a) Show that every set of n linearly independent vectors in V constitutes a basis of V.

(b) Show that every basis of V contains n (linearly independent) vectors.

[10] Let $\mathcal{A}$ be the subspace of $\mathbb{R}^4$ spanned by the columns of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 3 & 8 \\ 5 & 1 & -3 \\ -3 & -4 & -5 \end{pmatrix}$$

(a) find a basis of $\mathcal{A}$.

(b) What is the dimension of $\mathcal{A}$?

(c) Extend the basis of $\mathcal{A}$ to a basis of $\mathbb{R}^4$?