Math102.18, **Quiz #1**, Semester 042

Name:
ID #:

1. Find $\int x^8\sqrt{x^3-1}\ dx$. (3 points)
2. Find $\int \frac{\cos t}{1+2\sin t+\sin^2 t}dt$.
3. Find the sum $\sum_{k=3}^{n}\frac{4-k}{n-1}$. (4 points)

Good Luck,
Dr. Ibrahim Al-Rasasi

[1] Let $u = x^3-1 \Rightarrow du = 3x^2\,dx$

$\int x^6 \cdot x^2 \sqrt{x^3-1}\ dx$

$= \int (u+1)^2 \sqrt{u} \cdot \frac{1}{3}\,du$

$= \int (u^2+2u+1)\, u^{1/2} \cdot \frac{1}{3}\,du$

$= \frac{1}{3}\int u^{5/2} + 2u^{3/2} + u^{1/2}\ du$

$= \frac{1}{3}\left[\frac{2}{7}u^{7/2} + 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$

$= \frac{2}{21}(x^3-1)^{7/2} + \frac{4}{15}(x^3-1)^{5/2} + \frac{2}{9}(x^3-1)^{3/2} + C$

[2] $\int \frac{\cos t}{(1+\sin t)^2}\,dt$

Let $u = 1+\sin t \Rightarrow du = \cos t\,dt$

$= \int \frac{1}{u^2}\,du$

$= -\frac{1}{u} + C$

$= \frac{-1}{1+\sin t} + C$

[3] $\frac{1}{n-1}\sum_{K=3}^{n} 4-K$

$K = 3 \Rightarrow K-2 = 1$

Let $J = K-2 \Rightarrow K = J+2$

$K=3 \Rightarrow J=1$

$K=n \Rightarrow J = n-2$

$= \frac{1}{n-1}\cdot\sum_{J=1}^{n-2} 4-(J+2)$

$= \frac{1}{n-1}\cdot\sum_{J=1}^{n-2} 2-J$

$= \frac{1}{n-1}\left(\sum_{J=1}^{n-2} 2 - \sum_{J=1}^{n-2} J\right)$

$= \frac{1}{n-1}\left[2\cdot(n-2) - \frac{(n-2)(n-2+1)}{2}\right]$

$= (n-2)\left[\frac{2}{n-1} - \frac{1}{2}\right]$

$= \frac{2(n-2)}{n-1} - \frac{(n-2)}{2}$

$= (n-2)\,\frac{4-n+1}{2(n-1)}$

$= \frac{4(n-2) - (n-2)(n-1)}{2(n-1)}$

$= \frac{(n-2)(5-n)}{2(n-1)}$

$= \frac{-n^2+7n-10}{2(n-1)}$

$= \frac{-n^2+7n-10}{2(n-1)}$