

Math102.13. Quiz #1. Semester 042

Name:

ID#:

1. Find  $\int \frac{x}{\sqrt{1-16x^4}} dx$ . (3 points)
2. Find  $\int \frac{1}{2 \cot t - \sec t \csc t} dt$ . (3 points)
3. Find the sum  $\sum_{k=1}^{n-1} \left[ 3 \left( \frac{2k}{n} \right)^2 - \frac{2}{n-1} \right]$ . (4 points)

Good Luck,  
Dr. Ibrahim Al-Rasasi.

[1]  $\int \frac{x}{\sqrt{1-(4x^2)^2}} dx$

Let  $u = 4x^2 \Rightarrow du = 8x dx$

$$= \frac{1}{8} \int \frac{1}{\sqrt{1-u^2}} du$$

$$= \frac{1}{8} \sin^{-1} u + C$$

$$= \frac{1}{8} \sin^{-1}(4x^2) + C$$

[2]  $\int \frac{1}{2 \frac{\cos t}{\sin t} - \frac{1}{\cos t} \cdot \frac{1}{\sin t}} dt$

$$= \int \frac{1}{2 \cos^2 t - 1} dt$$

$$= \int \frac{\sin t \cos t}{2 \cos^2 t - 1} dt$$

$$= \frac{1}{2} \int \frac{\sin(2t)}{\cos(2t)} dt$$

$$= \frac{1}{2} \int \tan(2t) dt$$

Let  $u = 2t \Rightarrow du = 2 dt$

$$= \frac{1}{4} \int \tan u du$$

$$= \frac{1}{4} \ln |\sec u| + C$$

$$= \frac{1}{4} \ln |\sec(2t)| + C$$

[3]  $\sum_{k=1}^{n-1} \left[ \frac{12k^2}{n^2} - \frac{2}{n-1} \right]$

$$= \sum_{k=1}^{n-1} \frac{12k^2}{n^2} - \sum_{k=1}^{n-1} \frac{2}{n-1}$$

$$= \frac{12}{n^2} \sum_{k=1}^{n-1} k^2 - \frac{2}{n-1} \sum_{k=1}^{n-1} 1$$

$$= \frac{12}{n^2} \cdot \frac{(n-1)(n)(2(n-1)+1)}{6} - \frac{2}{n-1} \cdot (n-1)$$

$$= 2 \frac{(n-1)(2n-1)}{n} - 2$$

$$= 2 \left[ \frac{2n^2 - 3n + 1}{n} - 1 \right]$$

$$= 2 \cdot \frac{2n^2 - 4n + 1}{n} = 2 \left( 2n - 4 + \frac{1}{n} \right)$$

Done.