

King Fahd University of Petroleum and minerals
Department of Mathematics and Statistics
Math 455, **Exam III**, Semester 071

Jan. 12, 2008

Allowed Time: 2 hours

1. (4+2+4)
 - a. How many primitive roots does the prime 11 have? Find one.
 - b. Use your answer in part (a) to find a reduced residue system modulo 11.
 - c. How many solutions does the congruence $x^{14} \equiv 5 \pmod{11}$ have? Find them.
2. Let p be a prime and let h be the order of a modulo p . Show that $a^{h-1} + a^{h-2} + \dots + a + 1 \equiv 0 \pmod{p}$. (3 points)
3. Prove that if $p \equiv 1 \pmod{4}$ and $q = 2p + 1$ are primes, then 2 is a primitive root of q . (6 points)
4. Let p and $q = 6p + 1$ be two odd primes. Show that $\left(\frac{p}{q}\right) = \left(\frac{-1}{p}\right)$. (5 points)
5. Find all primes p for which the congruence $x^2 \equiv 17 \pmod{p}$ is solvable. (7 points)
6. Evaluate the Jacobi Symbol $\left(\frac{35}{123}\right)$. (4 points)
7. Let k be a positive integer. Find e if $7^e \parallel (7^k - 1)!$. (5 points)
8. Prove that $2 \leq d(n) < n$ for all integers $n > 2$. (5 points)
9. Let N be a given positive integer. Prove that there is only a finite number of positive integers x satisfying $\sigma(x) = N$. (5 points)
10. Let n be an even perfect number. Evaluate the product $\prod_{d|n} d$. (5 points)
11. Let n be a positive integer. Evaluate the sum $\sum_{d|n} \frac{\mu(d)\sigma(d)}{d(d)}$. (5 points)

All the best,
Dr. Ibrahim Al-Rasasi