King Fahd University of Petroleum and minerals
Department of Mathematics and Statistics
Math 455, **Exam II**, Semester 071
Dec.9, 2007
Allowed Time: 2 hours

1. Find the last two digits (units and tens digits) of the number 21^{1321}. (4 points)
2. Show that the number $1729 = 7\cdot 13\cdot 19$ is a Carmichael number. (4 points)
3. Show that no integer of the form $n = 5p$, where $p \neq 3$ is prime, is a pseudoprime to the base 3. (6 points)
4. Let p be an odd prime. (5+5 points)
   a. Prove that if $a \equiv 1 \bmod p$, then $a(a+1)(a+2)\cdots(a+(p-2)) \equiv -1 \bmod p$.
   b. Prove that $\frac{(5p)!}{5!\,p^5} \equiv -1 \bmod p$. (Hint: Use part a)
5. Characterize all positive integers n such that 4 does not divide $\phi(n)$. (5 points)
6. Let $S = \{\frac{a}{b} : 1 \le a \le b,\ \ (a,b) = 1,\ \ b \le 20\}$. Find the cardinality of S. (4 points)
7. Let $f(x)$ be a polynomial with integral coefficients having degree n. Show that if $x \equiv a \bmod m$ is a solution of $f(x) \equiv 0 \bmod m$, then $f(x) \equiv (x-a)g(x) \bmod m$ for some polynomial $g(x)$ of degree $n-1$. (4 points)
8. Let p be an odd prime and k be a positive integer. How many solutions does the congruence $x^{p-1} - 1 \equiv 0 \bmod p^k$ have. (5 points)
9. Solve: (3+5+5+5 points)
   a. $6x \equiv 12 \bmod 60$.
   b. $\begin{cases} x^3 - 2x + 1 \equiv 0 \bmod 7 \\ x^2 \equiv 4 \bmod 6 \end{cases}$
   c. $x^3 - x^2 - 2x + 2 \equiv 0 \bmod 5^3$.
   d. $2x^{21} - x^{15} + 3x^9 + 1 \equiv 0 \bmod 7$.

All the best,
Dr. Ibrahim Al-Rasasi