

King Fahd University of Petroleum and minerals
Department of Mathematics and Statistics
Math 455, **Exam I**, Semester 071
Nov. 3, 2007 (Shawwal 22, 1428)
Allowed Time: 90 minutes

1. (2+3+4=9 points)
 - a. Prove that if $(2a, a-2) = g$, then $g \mid 4$.
 - b. Find the number of decimal digits of the number 5^{55} .
 - c. Find a natural number n such that the interval $[n, n+100]$ contains no primes.
2. Show that the square of a number not divisible by 2 or 3 is of the form $12k+1$. (5 points)
3. Let p and q be two primes such that $p > q \geq 5$. Prove that either $3 \mid p+q$ or $3 \mid p-q$. (5 points)
4. Find $(455, 232)$ and express the answer as a linear combination of 455 and 232. (5 points)
5. Find $(2^{45} + 1, 2^{11} - 1)$. (4 points)
6. Let p_n denote the n^{th} prime. Prove that $p_n > 2n-1$ for all $n \geq 5$. (5 points)
7. Prove that an odd prime cannot be written as a sum of two positive cubes. (4 points)
8. Show that $2^{4n+2} + 1$ is composite for each positive integer n . (3 points)
9. Let m and n be two positive integers. Find all integer solutions of $nx - (n+1)y = m$. How many positive solutions does the equation have. (4 points)
10. Use Fermat's Factorization method to find nontrivial factors for the number 1537. (4 points)
11. Find the remainder when the number $2^{61} + 5^{30}$ is divided by 9. (4 points)
12. What are the possible units digits of a cube. (3 points)
13. Let m and n be two positive integers greater than 1 such that $(m, n) = 1$. Prove that if $\{r_1, r_2, \dots, r_m\}$ is a complete residue system modulo m and $\{s_1, s_2, \dots, s_n\}$ is a complete residue system modulo n , then the set $\{nr_i + ms_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ forms a complete residue system modulo mn . (5 points)

All the best,
Dr. Ibrahim Al-Rasasi