

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 232, **Exam 1**, Semester 061
Monday, Nov. 6, 2006
Allowed Time: 90 minutes

1. Consider the following if-then statement: (6+4 points)

"If $f'(x) = 0$ for all x in an interval (a,b) , then f is constant on (a,b) ."

- a. Rewrite the above statement using the following words:
 - i. Imply
 - ii. Only if
 - iii. Necessary
 - b. Give the **contrapositive** of the above statement.
2. (2+5+5 points)
- a. TRUE or FALSE (Justify your answer). If $2+5=4$, then $1+1=2$.
 - b. Give a **counterexample** to the following statement: "For every real numbers x and y , if $x < y$, then $x^2 < y^2$."
 - c. Let $A = \{2n+3 : n \in \mathbb{Z}\}$ and $B = \{3n+2 : n \in \mathbb{Z}\}$ where \mathbb{Z} is the set of integers. Show that $A \not\subseteq B$.
3. Write the truth table for the propositional form $(\neg P \Rightarrow Q) \wedge R$. (5 points)
4. Give an example of a biconditional which is a contradiction. (5 points)
5. Let P, Q, R and S be propositions. Give a two-column proof for each of the following statements: (10+10 points)
- a. $P \Rightarrow Q, R \Rightarrow S, P \vdash Q \vee S$.
 - b. $(Q \vee P) \Rightarrow (R \wedge S) \vdash Q \Rightarrow R$.
6. Write the **negation** of the following statement: (8 points)

"For every positive real number M , there exists a real number N such that if $x > N$, then $f(x) > M$ ".

7. **Prove** each of the following statements and **name** the method of proof that you used. (10+10 points)
- a. If n is an odd integer, then 4 divides $n^2 + 11$.
 - b. If n is an integer, then $n^2 + 3n + 2$ is an even integer.
8. Let A, B, C and D be sets. (10+10 points).
- a. Prove that if $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$.
 - b. Prove $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$.

All the best,
Dr. Ibrahim Al-Rasasi.