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Math 201- Review Problems
(60 Problems)

1. Sketch the graph of the polar equation $r = 2\sin^2(\frac{\theta}{2})$. **Hint**: First simplify using a suitable trigonometric identity.
2. Sketch the graph of the polar equation $r = |\cos\theta|$.
3. Sketch and compare the graphs of the polar equations $r = 1 - \cos\theta$ and $r = -1 - \cos\theta$.
4. Convert the polar equation $r = \cos(3\theta)$ to a rectangular equation.
5. Test for symmetry of the graph of the polar curve $r^2 = \cos(2\theta)$ about the polar axis, the line $\theta = \frac{\pi}{2}$, and the pole. Check your answer by graphing the curve. **Answer**: It is symmetric about all.
6. Describe the graph of the parametric curve given by $x = \sin^2 t, y = \cos^2 t$, $0 \le t \le \pi$. **Answer**: The line segment $y = 1 - x$, $0 \le x \le 1$.
7. Find the equation of the tangent line to the parametric curve given by $x = \sin^2 t, y = \cos^2 t$, $0 \le t \le \pi$, at the point where $t = \frac{\pi}{4}$. **Answer**: $y = 1 - x$.
8. Find the slope of the tangent line to the polar curve $r = \ln\theta$ at the point where $\theta = e$. **Answer**: $\frac{e\cos e + \sin e}{\cos e - e\sin e}$.
9. Find the arc length of the parametric curve $x = t - \sin t, y = 2 - \cos t$, $0 \le t \le \frac{\pi}{2}$. **Answer**: $4 - 2\sqrt{2}$.
10. Find the area enclosed by one leaf of the rose $r = \sin(5\theta)$. **Answer**: $\frac{\pi}{20}$.
11. Describe the graph of the equation $3x^2 + 3y^2 + 3z^2 + 6y - 2z - 1 = 0$. **Answer**: A sphere with center (0, -1, 1/3) and radius $\frac{\sqrt{13}}{3}$.
12. Let **v**$=<a, b+2c, 3>$ and **w**$=<2b+1, 2a-b, 4c>$ be two vectors in 3-space. Find the values of a, b and c if **v**=**w. Answer**: $a = \frac{1}{2}, b = \frac{-1}{4}, c = \frac{3}{4}$.
13. What is the geometric effect of multiplying a vector by a nonzero constant.
14. Let v be a vector in 3-space and k be a scalar. Prove that $\|kv\| = |k|\|v\|$.

15. If **u** and **v** are orthogonal to **w**, Is **u** + **v** orthogonal to **w**. Explain your answer.
16. Find the angle between the vectors **v** and 2**v** and the angle between the vectors **v** and –**v**. **Answer**: 0, π .
17. Show that the vector **u** =<2, -1, 5> is not a scalar multiple of the vector **v** =<1, -2, 4>.
18. Find a unit vector orthogonal to the vector with initial point $P=(1,-1,0)$ and terminal point $Q=(2,1,-1)$. **Answer**: A possible answer is $<\frac{2}{\sqrt{5}},\frac{-1}{\sqrt{5}},0>$. There are other possibilities.
19. If $\theta=\frac{\pi}{3}$ is the angle between the two unit vectors **u** and **v**, then find the dot product of **u** and **v**. **Answer**: ½.
20. If **w**=$<a,b,a>$ is a unit vector, then what is the value of a^2+b^2. **Answer**: $1-a^2$.
21. TRUE or FALSE (Justify your answer): **u**×(**v**×**w**)=(**u**×**v**)×**w**.
22. Determine whether the point (1, -1, 3) lies on the line $x=t, y=3+2t, z=1-2t$, $-\infty<t<+\infty$. **Answer**: The point does not lie on the line.
23. Determine whether the point (0, 3, -5) lies on the plane $5x+6y+7z=-17$. **Answer**: The point lies on the plane.
24. Describe the following parametric curve in 3-space:
$$x=-1+t, y=1+2t, z=3t,\ 0\le t\le 2.$$
Answer: It is the line segment joining the points (-1, 1, 0) and (1, 5, 6)
25. Find the distance between the planes $ax+by+cz=d$ and $ax+by+cz=e$. **Answer**: $\frac{|e-d|}{\sqrt{a^2+b^2+c^2}}$.
26. Find the equation of the plane through (1, 1, 1) that is perpendicular to the line $x=1+t, y=2t, z=-1-t$. **Answer**: $x+2y-z=2$.
27. Find the point(s) of intersection between the plane $x+y-z=1$ and the line $x=1+2t, y=-1+t, z=2+4t$. **Answer**: (-5, -4, -10).
28. Identify the surface $x=y^2+z^2$. **Answer**: A circular paraboloid with vertex (0, 0, 0) and with the $x-axis$ as its axis.
29. Identify the surface $x^2=y^2+z^2$. **Answer**: A circular cone with vertex (0, 0, 0) and with the $x-axis$ as its axis.
30. Convert the equation $x^2+y^2=1$ to an equation in spherical coordinates. **Answer:** $\rho=\csc\phi$.
31. Find the domain and the range of $f(x,y)=x^3$. Sketch the graph of f. **Answer**: Domain={ (x,y) in 2-space: x and y are arbitrary real numbers}, Range= $(-\infty,+\infty)$.
32. Find and sketch the domain of $f(x,y)=\frac{x+y}{xy-1}$. **Answer**: Domain = $\{(x,y):$ $y\ne\frac{1}{x}\}$, i.e., all points in 2-space not lying on the hyperbola $y=\frac{1}{x}$.

33. Find and sketch the domain of $f(x,y)=\dfrac{x+\sqrt{y}}{xy-1}$. **Answer**: Domain = $\{(x,y): y \neq \dfrac{1}{x}, y \geq 0\}$, i.e., all points in the upper half- plane (including the $x-axis$) of 2-space that are not lying on the hyperbola $y=\dfrac{1}{x}$.

34. Find the limit $\lim_{(x,y)\to(1,1)} \dfrac{xy}{(x-1)^2+(y-1)^2}$. **Answer**: $+\infty$.

35. Find the limit $\lim_{(x,y)\to(1,1)} \dfrac{\sin(x^2-1)}{x^2y^2+x^2-y^2-1}$. **Answer**: $\dfrac{1}{2}$.

36. Find the limit $\lim_{(x,y)\to(0,0)} \dfrac{x^4 y}{x^8+y^2}$. **Answer**: DNE.

37. Find the region of continuity of $f(x,y)=\sin(x^2-y^3)$. **Answer**: It is continuous everywhere in 2-space.

38. Let $f(x,y)=x^y$. Find (i) $f_x(x,y)$ and (ii) $f_y(x,y)$. **Answer**: (i) yx^{y-1}, (ii) $x^y \ln x$.

39. Let $f(x,y)=\dfrac{1}{(x^2+3y^2)^2}$. Show that $xf_x(x,y)+yf_y(x,y)=-4f(x,y)$.

40. Let $f(x,y,z)=\dfrac{x+z}{y+z}$.
    a. Find the local linear approximation of f at (0, 0, 1). **Answer**: $L(x,y,z)=1+x-y$.
    b. Approximate $f(0.02, 0.01, 1.01)$. **Answer**: 1.01.

41. Let $z=u^2-\sqrt{v}, u=\dfrac{x}{1+\sin y}, v=\dfrac{y}{1+\sin x}, y=e^{-x^2}$. Find $\dfrac{dz}{dx}|_{x=0}$. **Answer:** 1/2.

42. If $\nabla f(2,-1) = <1,1>$ and $D_u f(2,-1)=1$, then find the unit vector **u**. **Answer:** Two possible answers: **u**=**i** or **u**=**j**.

43. Find the critical points of $f(x,y)=\dfrac{1}{x}+\dfrac{1}{y}$. **Answer:** None.

44. Locate the relative extrema and saddle points, if any, of $f(x,y)=-\sqrt{x^2+y^2}$. **Answer:** (0,0) is the only critical point as $f_x(0,0)$ DNE. Test fails. From the graph of f, f has an absolute maximum at (0, 0).

45. Locate the relative extrema and saddle points, if any, of $f(x,y)=x^2y^2$. **Answer:** (0,0) is the only critical point. Test fails. Since $x^2y^2 \geq 0$, then f has an absolute minimum at (0, 0).

46. Locate the relative extrema and saddle points, if any, of $f(x,y)=x^4-y^4$. **Answer:** (0,0) is the only critical point. Test fails. f has a saddle point at (0, 0), from the definition of a saddle point.

47. Locate the relative extrema and saddle points, if any, of $f(x,y)=x\sin y$. **Answer:** Critical points are $(0,n\pi)$, where n is an integer. f has saddle points at these critical points.
48. Locate the relative extrema and saddle points, if any, of $f(x,y)=x^3-y^3+xy$. **Answer:** Critical points are (0, 0) and (1/3, -1/3). f has saddle points at these critical points.
49. Locate the relative extrema and saddle points, if any, of $f(x,y)=xy^2-4x-y+1$. **Answer:** Critical points are (1/4, 2) and (-1/4, -2). f has saddle points at these critical points.
50. Find a unit vector in 2-space such that the sum of its components is as large as possible. **Answer:** $<\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}>$.
51. Use Lagrange Multipliers to find three numbers whose sum is 27 and such that the sum of their squares is as small as possible. **Answers:** The numbers are 9, 9, 9.
52. Maximize $f(x,y)=y-x$ subject to the constraint $x^2+y^2=1$. **Answer:** The maximum value of f is $\sqrt{2}$. It occurs at the point $(\frac{-\sqrt{2}}{2},\frac{\sqrt{2}}{2})$.
53. Find the volume of the solid enclosed between the surface $z=e^{-y^2}$ and the triangular region R with vertices (0, 0), (0, 4) and (1, 4). **Answer:** $\frac{1}{8}(1-e^{-16})$.
54. Evaluate $\iint_R x\,dA$ where R is the region enclosed between the curves $x^2+y^2=2$ and $y=x^2$. **Answer:** $\int_{-1}^{1}\int_{x^2}^{\sqrt{2-x^2}} x\ dydx=0$.
55. Find the volume of the solid enclosed between the plane $x+z=4$ and the triangular region R with vertices (1, 1), (-1, -1) and (3, -3). **Answer:** $V=\int_{-1}^{1}\int_{-\frac{1}{2}x-\frac{3}{2}}^{x}(4-x)dydx+\int_{1}^{3}\int_{-\frac{1}{2}x-\frac{3}{2}}^{-2x+3}(4-x)dydx$.
56. Use double integrals to find the area of the region enclosed by the curves $x=\sqrt{y}, x=1, y=3$. **Answer:** $A=\int_1^3\int_1^{\sqrt{y}}dxdy=\frac{2(3\sqrt{3}-4)}{3}$.
57. Evaluate $\iint_R \sin(\frac{y}{x})dA$ where R is the region enclosed by the curves $y=0, y=x^3, x=\sqrt{\pi}, x=\sqrt{2\pi}$. **Answer:** $\frac{\pi}{2}$.
58. Evaluate $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\sqrt{1-x^2-y^2}\,dxdy$. **Answer:** $\frac{2\pi}{3}$.
59. Use double integrals to find the volume of the solid in the first octant bounded by the planes $x=0, y=0, z=0, x+y+z=1$. **Answer:** $\int_0^1\int_0^{1-x}(1-x-y)dydx=\frac{1}{6}$.

60. Use triple integrals to find the volume of the solid in the first octant bounded by the planes $x = 0, y = 0, z = 0, x + y + z = 1$. **Answer:** $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dzdydx = \frac{1}{6}$.