1. An estimate of the area under the graph of $f(x) = 16 - x^2$ from $x = 0$ to $x = 4$ using four approximating rectangles and left endpoints is

(a) 50

(b) 40

(c) 30

(d) 20

(e) 45

2. The value of the limit $\lim_{n \to +\infty} \sum_{i=1}^{n} \left(\frac{4i}{n^2} + \frac{3}{n} \right)$ is equal to

(a) 5

(b) 7

(c) 1

(d) -4

(e) -8

3. The integral $\int \frac{\cos\left(\frac{\pi}{x^2}\right)}{x^3}\,dx$ is equal to

(a) $-\frac{1}{2\pi}\sin\left(\frac{\pi}{x^2}\right)+C$

(b) $\frac{1}{2\pi}\sin\left(\frac{\pi}{x^2}\right)+C$

(c) $-\frac{1}{2\pi}\sin\left(\frac{\pi}{x}\right)+C$

(d) $\frac{1}{x^2}\sin\left(\frac{\pi}{x}\right)+C$

(e) $\frac{1}{x^2}\sin\left(\frac{\pi}{x^2}\right)+C$

4. The integral $\int \frac{-4x}{\sqrt{1-4x^2}}\,dx$ is equal to

(a) $\sqrt{1-4x^2}+C$

(b) $-\frac{1}{4}\sqrt{1-4x^2}+C$

(c) $\frac{1}{\sqrt{1-4x^2}}+C$

(d) $\frac{-8}{\sqrt{1-4x^2}}+C$

(e) $16\sqrt{1-4x^2}+C$

5. The area of the region bounded by the curves $y = e^x, y = x, x = 0$ and $x = 1$ is

(a) $e - \frac{3}{2}$

(b) $e + \frac{1}{2}$

(c) e

(d) $3 - e$

(e) $e + 1$

6. By interpreting the integral $\int_{-2}^{2} (3 + \sqrt{4 - x^2})\, dx$ in terms of areas, its value is equal to

(a) $12 + 2\pi$

(b) $6 + 2\pi$

(c) $6 + \pi$

(d) $12 + \pi$

(e) $6 + 4\pi$

7. The value of the integral $\int_1^8 \frac{1+\sqrt[3]{x}}{\sqrt[3]{x^2}}\,dx$ equals

(a) $\frac{15}{2}$

(b) $\frac{45}{2}$

(c) $\frac{7}{2}$

(d) $\frac{25}{2}$

(e) $\frac{17}{2}$

8. The value of the integral $\int_1^{e^4} \frac{\sqrt{\ln x}}{x}\,dx$ is equal to

(a) $\frac{16}{3}$

(b) $\frac{8}{e^4}$

(c) 8

(d) $\frac{2}{e^4}$

(e) 0

9. The value of the integral $\int_{-3}^{3} \sin(x^5)\, dx$ is equal to

(a) 0

(b) $2\cos(243)$

(c) 6

(d) -3

(e) 1

10. The area of the region bounded by the curves $y^2 = 4 - x$ and $x + 2y = 1$ is given by the definite integral

(a) $\int_a^b (3 + 2y - y^2)dy,$ where $a + b = 2$

(b) $\int_a^b (3 + 2y - y^2)dy,$ where $a + b = 5$

(c) $\int_a^b (y^2 - 2y - 3)dy,$ where $a + b = 4$

(d) $\int_a^b \left[\sqrt{4-x} - \frac{1}{2}(1-x)\right] dx,$ where $a + b = -2$

(e) $\int_a^b \left[\frac{1}{2}(1-x) - \sqrt{4-x}\right] dx,$ where $a + b = 8$

11. The area of the region enclosed by the curves $y = \sin x, y = \cos x, x = 0$ and $x = \pi$ is

(a) $2\sqrt{2}$

(b) $2\sqrt{2} - 1$

(c) $2\sqrt{2} + 1$

(d) $\sqrt{2} + 2$

(e) $-\sqrt{2}$

12. If f' is continuous on $[1, 3]$, then $\int_1^3 f'(x)dx =$

(a) $f(3) - f(1)$

(b) $f'(3) - f'(1)$

(c) $f(2)$

(d) $f(1) - f(3)$

(e) $f(3) + f(1)$

13. When expressing the limit

$$\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{\pi}{3n} \cot\left(\frac{\pi}{6} + \frac{i\pi}{3n}\right)$$

as a definite integral, it becomes

(a) $\displaystyle\int_{\pi/6}^{\pi/2} \cot x \, dx$

(b) $\displaystyle\int_{\pi/6}^{\pi/3} \cot x \, dx$

(c) $\displaystyle\int_{\pi/6}^{\pi/2} \cot\left(\frac{\pi}{6} + x\right) dx$

(d) $\displaystyle\int_{\pi/6}^{\pi/4} x \cot\left(\frac{\pi}{6} + x\right) dx$

(e) $\displaystyle\int_{\pi/3}^{\pi/2} \cot x \, dx$

14. If f is continuous and $\displaystyle\int_3^5 f(x)\, dx = 8$, then $\displaystyle\int_0^1 f(2x+3)\, dx =$

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

15. The velocity (in meters per second) of a particle moving along a line is given by $V(t) = 3t^2 - 12t + 9$. The distance traveled between $t = 0$ and $t = 2$ is

(a) 6 meters

(b) 8 meters

(c) 4 meters

(d) 9 meters

(e) 5 meters

16. The integral $\int \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\, d\theta$ is equal to

(a) $-\cos\theta + C$

(b) $\tan\theta + C$

(c) $\cos\theta + C$

(d) $3\sin\theta + C$

(e) $-\sin\theta + C$

17. If $f(x) = \int_1^{x^3+3x} (t^3+1)^{20}\, dt$, then $f'(0)$ equals

(a) 3

(b) 0

(c) $\frac{1}{7}$

(d) $\frac{3}{20}$

(e) $\frac{20}{3}$

18. $\lim_{x \to 0} \left(1 + \frac{1}{2}x\right)^{\frac{1}{2x}} =$

(a) $\sqrt[4]{e}$

(b) $\sqrt{e}$

(c) e

(d) e^2

(e) e^4

19. The base of a solid S is enclosed by the curves $y = x^2, y = 0$ and $x = 2$. If the cross-sections of S perpendicular to the x-axis are squares, then the volume of S is equal to

(a) $\frac{32}{5}$

(b) $\frac{16}{3}$

(c) $\frac{19}{4}$

(d) 13

(e) 4

20. If the region enclosed by the curves $y = \frac{1}{2}x$ and $y = \sqrt{x}$ is rotated about the line $x = -1$, then the volume of the solid is given by

(a) $\pi \int_0^2 [(2y+1)^2 - (y^2+1)^2] dy$

(b) $\pi \int_0^2 [(2y-1)^2 - (y^2-1)^2] dy$

(c) $\pi \int_0^4 \left[\left(\frac{1}{2}x + 1 \right)^2 - (\sqrt{x}+1)^2 \right] dx$

(d) $\pi \int_0^4 \left[\left(\frac{1}{2}x - 1 \right)^2 - (\sqrt{x}-1)^2 \right] dx$

(e) $\pi \int_0^2 [2y - y^2 - 1]^2 dy$