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Section 7.5: <u>Strategy for Integration</u>

So far we have studied seven techniques of integration:

1. u-Substitution
2. Integration by Parts
3. Trigonometric Integrals
4. Trigonometric Substitution
5. Partial Fractions (for Rational Functions)
6. Radical Expressions (or Mixed Radicals)
7. Rational Expressions of sines and cosines

In general it is not difficult to decide which technique to use to solve a given integration problem. Usually the form of the integrand will direct you to the suitable technique. You are advised to read the general guidelines given in the textbook (pp. 483-489). These guidelines will help you in deciding the suitable technique to solve integration problems. You also need to remember that **"Practice is the best teacher"** in learning mathematics.

In what follows we give some examples with brief solutions.

Examples: Find the following integrals.

1. $\int (1+\sqrt{x})^8 dx$.
2. $\int \frac{1}{e^x - e^{-x}} dx$.
3. $\int \frac{\ln(\tan x)}{\sin x \cos x} dx$.
4. $\int (x \sin^2 x \cos x) dx$.
5. $\int \frac{\sqrt{1+\ln x}}{x \ln x} dx$.
6. $\int_{-\pi}^{\pi} (\sin^9 x) dx$.
7. $\int \frac{1}{x + \sqrt[3]{x}} dx$.
8. $\int \frac{x^3+1}{x^3 - x^2} dx$.
9. $\int \frac{1}{\sqrt{4x^2 - 4x - 3}} dx$.

10. $\int \frac{\sin^{-1}(\sqrt[3]{x})}{\sqrt[3]{x^2}}dx$.

11. $\int \sqrt{\frac{1+x}{1-x}}dx$.

Brief Solutions:

1. u-substitution is enough. Let $u = 1+\sqrt{x}$.
2. Multiply the numerator and denominator by e^x and then let $y = e^x$.
3. u-substitution is enough. Let $u = \ln(\tan x)$.
4. Use integration by parts: let $u = x$, $dv = (\sin^2 x \cos x)dx$.
5. Use power substitution: let $u^2 = 1+\ln x$ $(u = \sqrt{1+\ln x})$.
6. The integration of an odd function over a symmetric interval is zero.
7. Let $x = u^3$ $(u = \sqrt[3]{x})$. The final answer is $\frac{3}{2}\ln(\sqrt[3]{x^2}+1)+C$.
8. Divide first using long division and then decompose into partial fractions.
9. First complete the square of the radicand, then make the substitution $y = 2x-1$, and then use the trigonometric substitution $y = 2\sec\theta$.
10. First make the substitution $y = \sqrt[3]{x}$ and then use integration by parts.
11. First multiply by $\sqrt{\frac{1+x}{1+x}}$ and simplify:

$$\int \sqrt{\frac{1+x}{1-x}}\sqrt{\frac{1+x}{1+x}}dx = \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}dx = \sin^{-1}x - \sqrt{1-x^2} + C$$