King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Calculus II
FINAL EXAM
Semester II, Term 082
Monday, June 22, 2009

EXAM COVER

Number of versions: 4
Number of questions: 28
Number of Answers: 5 per question

This exam was prepared using mcqs
For questions send an email to Dr. Ibrahim Al-Lehyani (iallehyani@kaau.edu.sa)

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Calculus II
FINAL EXAM
Semester II, Term 082
Monday, June 22, 2009
Net Time Allowed: 180 minutes

MASTER VERSION

1. The average value of the function $f(x) = \cos^4 x \sin x$ over $[0, \pi]$ is

(a) $\dfrac{2}{5\pi}$

(b) $\dfrac{2}{\pi}$

(c) $\dfrac{2}{5}$

(d) $\dfrac{5\pi}{2}$

(e) $\dfrac{2}{3}$

2. $\displaystyle\int \tan^4 x \, dx =$

(a) $\dfrac{1}{3}\tan^3 x - \tan x + x + c$

(b) $\dfrac{1}{3}\tan^3 x + \tan x \sec x + c$

(c) $\ln|\sec x + \tan x| + \tan^3 x + c$

(d) $\ln|\csc x - \sec x| + \tan x + c$

(e) $\sec^3 x + 3\sec^2 x + c$

3. The series $\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$ is

(a) divergent

(b) convergent to 0.1

(c) convergent to 100

(d) convergent to 0.01

(e) convergent to 0.001

4. For what values of p, is the series $\sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^{p-4}}$ convergent?

(a) $p > 4$

(b) $p > 1$

(c) $p \geq 4$

(d) $p < 4$

(e) $p \leq 4$

5. $\displaystyle\int \frac{1}{x^2\sqrt{25-x^2}}\,dx =$

(a) $\dfrac{-1}{25}\dfrac{\sqrt{25-x^2}}{x} + c$

(b) $\dfrac{1}{125}\dfrac{\sqrt{25-x^2}}{x} + c$

(c) $\dfrac{-1}{5}\dfrac{\sqrt{25-x^2}}{x} + c$

(d) $\dfrac{1}{5}\dfrac{\sqrt{25-x^2}}{x} + c$

(e) $\dfrac{\sqrt{25-x^2}}{x} + c$

6. The radius of convergence of the power series $\displaystyle\sum_{n=0}^{+\infty} \frac{(x-1)^n}{3^n}$ is

(a) $R = 3$

(b) $R = 1$

(c) $R = \dfrac{1}{3}$

(d) $R = \infty$

(e) $R = 0$

7. $\displaystyle\int \frac{x^2 - x + 6}{x^3 + x}\, dx =$

(a) $6\ln|x| - \frac{5}{2}\ln(x^2+1) - \tan^{-1}(x) + c$

(b) $6\ln|x| - \frac{5}{2}\ln(x^2+1) + \tan^{-1}(x) + c$

(c) $6\ln|x| - \frac{5}{2}\ln(x^2+1) - \sin^{-1}(x) + c$

(d) $\ln|x| - \frac{1}{2}\ln(x^2+1) - \tan^{-1}(x) + c$

(e) $-\frac{5}{2}\ln(x^2+1) - \tan^{-1}(x) + c$

8. $\displaystyle\int_1^3 \frac{\sqrt{x}}{x^2 + x} dx =$

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) 2π

(d) 5π

(e) $\frac{5\pi}{4}$

9. The area of the region enclosed by the graphs of $y = x - 1$ and $x = (y-1)^2$ is equal to

(a) $\frac{9}{2}$

(b) 9

(c) 3

(d) $\frac{8}{3}$

(e) 8

10. The improper integral $\int_0^{\pi/2} \frac{\cos x}{1 - \sin x}\, dx$

(a) diverges

(b) converges and has the value 0

(c) converges and has the value $\frac{\pi}{4}$

(d) converges and has the value π

(e) converges and has the value $\frac{\pi}{2}$

11. The area of the surface obtained by rotating the curve $y = x^3, \quad 0 \leq x \leq 1$ about the x-axis is

(a) $\frac{\pi}{27}(10\sqrt{10} - 1)$

(b) $\frac{\pi}{27}(145\sqrt{145} - 1)$

(c) $\frac{\pi}{18}(10\sqrt{10} - 1)$

(d) $\frac{\pi}{18}(145\sqrt{145} - 1)$

(e) $\frac{\pi}{27}$

12. The sum of the series $\sum_{n=1}^{\infty}[\frac{3}{n(n+1)} + \frac{1}{2^n}]$ is equal to

(a) 4

(b) 3

(c) 2

(d) 1

(e) 5

13. The volume of the solid obtained by rotating the region enclosed by the curves $y = \cosh x$, $y = \sinh x$, $x = 0$ and $x = 5$, about the x-axis, is (Hint: $\cosh x - \sinh x > 0$)

(a) 5π

(b) $\dfrac{\pi}{5}$

(c) $\dfrac{5\pi}{2}$

(d) π

(e) $\dfrac{\pi}{5} - 1$

14. Which one of the following statements is TRUE about the alternating series $\sum_{n=1}^{+\infty} (-1)^{n-1} a_n$, where $a_n = \dfrac{2n}{3n+1}$?

(a) The series is divergent

(b) The series is absolutely convergent

(c) The series is conditionally convergent

(d) $a_{n+1} \leq a_n$for all n

(e) $\sum_{n=1}^{+\infty} (-1)^{n-1} a_n = \dfrac{2}{3}$

15. How many terms of the series $\sum_{n=1}^{+\infty} \frac{(-1)^{n-1}}{n^3}$ do we need to add in order to ensure that the sum is accurate to within 0.001? (minimum number of terms)

(a) 9

(b) 10

(c) 100

(d) 99

(e) 1000

16. By applying the Ratio Test to the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$, we conclude that the

(a) test fails

(b) series is convergent

(c) series is absolutely convergent

(d) series is conditionally convergent

(e) series is divergent

17. The interval of convergence of the power series $\sum_{n=1}^{+\infty} \frac{(2x-3)^n}{n4^{2n}}$ is

(a) $\left[-\frac{13}{2}, \frac{19}{2}\right)$

(b) $\left(-\frac{13}{2}, \frac{19}{2}\right)$

(c) $\left(-\frac{13}{2}, \frac{19}{2}\right]$

(d) $\left[-\frac{13}{2}, \frac{19}{2}\right]$

(e) $\left[\frac{13}{2}, \frac{19}{2}\right)$

18. By applying the Root Test to the series $\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{3^{1+3n}}$, we conclude that the

(a) series is divergent

(b) test fails

(c) series is convergent

(d) series is absolutely convergent

(e) series is conditionally convergent

19. The Taylor series of $f(x) = \dfrac{1}{x}$ about $x = 2$ is

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^n$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x-2)^n$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x-2)^{n+1}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x+2)^n$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x+2)^n$

20. The coefficient of x^4 in the Maclaurin series of $\cos^2 x$ is

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) 2

(d) $\frac{1}{2}$

(e) $\frac{1}{4}$

21. Let us consider the series $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$. Then the integral test

(a) implies that the series converges

(b) is not applicable because of the continuity condition

(c) implies that the series diverges

(d) is not applicable because of the decreasing condition

(e) is not applicable because of the positivity condition

22. Let us consider the sequence $\{\tan^{-1}(-3n)\}$. Then the sequence

(a) converges and its limit is $-\frac{\pi}{2}$

(b) converges and its limit is 0

(c) converges and its limit is $\frac{\pi}{2}$

(d) diverges

(e) converges and its limit is -1

23. The sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$ is

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{3}}{2}$

(e) $\sqrt{3}$

24. $\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1+t^3}\,dt$

(a) is equal to 3

(b) is equal to 4

(c) is equal to -3

(d) is equal to 0

(e) does not exist

25. The volume of the solid obtained by rotating the region enclosed by $y = \dfrac{1}{x^2+2x+2}$, $x = 1$, $x = 2$, about the line $x = -1$ is

(a) $\pi \ln 2$

(b) $2\pi \ln 2$

(c) $\pi \ln 2 - 2\pi(\tan^{-1} 3 - \tan^{-1} 2)$

(d) $2\pi(\tan^{-1} 3 - \tan^{-1} 2)$

(e) $\pi(\tan^{-1} 2 - \tan^{-1} 3)$

26. $\displaystyle\int_0^{\pi^2/4} \cos\sqrt{x}\, dx =$

(a) $\pi - 2$

(b) $\dfrac{\pi}{2} - 1$

(c) $\dfrac{\pi}{2} - \dfrac{1}{2}$

(d) $\dfrac{\pi^2}{4} - 1$

(e) $\dfrac{\pi}{4} - 1$

27. The arc length of the curve $y = x^2 - \frac{1}{8}\ln x, \quad 1 \leq x \leq 3$ is equal to

(a) $8 + \frac{1}{8}\ln 3$

(b) $3 + \frac{1}{3}\ln 8$

(c) $8 + \ln 3$

(d) $3 + \ln 8$

(e) $-8 + \frac{1}{8}\ln 3$

28. The power series representation for the function $f(x) = \frac{x}{(x-2)^2}$ is (Hint: You may use differentiation)

(a) $\sum_{n=0}^{\infty} \frac{n+1}{2^{n+2}} x^{n+1}$

(b) $\sum_{n=0}^{\infty} \frac{n+1}{2^{n+2}} x^{n}$

(c) $\sum_{n=0}^{\infty} \frac{n+1}{2^{n+1}} x^{n+1}$

(d) $\sum_{n=0}^{\infty} \frac{n}{2^{n}} x^{n}$

(e) $\sum_{n=0}^{\infty} \frac{n+1}{2} x^{n+1}$