King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Summer (073)
Tuesday 26/8/2008
Net Time Allowed: 180 minutes

MASTER VERSION

1. The area of the region enclosed by the curves $y = 2x^2 - 1$ and $y = -2x - 1$ is equal to

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

(e) $\frac{2}{5}$

2. If the region enclosed by the curves $y = \sin x$ and $y = 0$ between $x = 0$ and $x = \pi$ is revolved about the $y-axis$, then the volume of the solid generated is equal to

(a) $2\pi^2$

(b) π^2

(c) $\frac{\pi^2}{2}$

(d) $\frac{\pi^2}{4}$

(e) $4\pi^2$

3. The series $\sum_{n=1}^{\infty} \left(\frac{n^3+1}{2n^3+n} \right)^n$

(a) Converges by the Root test

(b) Diverges by the Root test

(c) Diverges by the Ratio test

(d) is a convergent geometric series

(e) Diverges by the limit comparison test

4. $\sum_{n=0}^{\infty} \frac{(-1)^n + 2^{n+1}}{3^n} =$

(a) $\frac{27}{4}$

(b) $\frac{9}{4}$

(c) $\frac{11}{4}$

(d) $\frac{31}{4}$

(e) $\frac{23}{4}$

5. $\displaystyle\int \frac{\sqrt{1-x^2}-x}{\sqrt{1-x^2}}\ dx =$

(a) $x+\sqrt{1-x^2}+c$

(b) $x-\sin^{-1}x+c$

(c) $x-\frac{1}{2}\sqrt{1-x^2}+c$

(d) $x+\sin^{-1}x+c$

(e) $x+\frac{3}{2}\sqrt{1-x^2}+c$

6. $\displaystyle\int_0^{\pi} |\sin 2x|\ dx =$

(a) 2

(b) 0

(c) -1

(d) 1

(e) $\frac{1}{2}$

7. The set of all values of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-1)^{5p-1}}$ converges, is given by the interval

(a) $\left(\frac{1}{5}, \infty\right)$

(b) $(1, \infty)$

(c) $[1, \infty)$

(d) $\left[\frac{1}{5}, \infty\right)$

(e) $(0, \infty)$

8. The area of the surface obtained by rotating the curve $y = \sqrt{3 - x^2}$, $0 \leq x \leq 1$, about the x-axis is equal to

(a) $2\pi\sqrt{3}$

(b) $\frac{4\pi\sqrt{3}}{3}$

(c) $\frac{4\sqrt{3}}{2}$

(d) $6\pi\sqrt{3}$

(e) $\frac{\pi\sqrt{3}}{6}$

9. $\displaystyle\int \tan^{-1}\left(\frac{1}{x}\right)\,dx =$

(a) $x\tan^{-1}\left(\frac{1}{x}\right) + \ln\sqrt{1+x^2} + c$

(b) $\frac{1}{2}\,x\tan^{-1}\left(\frac{1}{x}\right) + \ln\sqrt{1+x^2} + c$

(c) $(x+1)\tan^{-1}\left(\frac{1}{x}\right) + c$

(d) $x\tan^{-1}\left(\frac{1}{x}\right) - \ln\left(1+x^2\right) + c$

(e) $\left(\frac{1}{x^2}\right)\tan^{-1}\left(\frac{1}{x}\right) + \ln\sqrt{1+x^2} + c$

10. The series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt[3]{n^4+5}}$

(a) Converges by the comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

(b) Converges by the comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(c) Converges by the integral test

(d) Diverges by the test for divergence

(e) Diverges by the comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$

11. $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{5}{2}}}\ dx =$

(a) $\dfrac{4}{3}$

(b) $\dfrac{7}{3}$

(c) $\dfrac{\sqrt{3}}{3}$

(d) $\dfrac{1}{3}$

(e) $\sqrt{3}$

12. The radius of convergence R and the interval of convergence I of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2x-3)^n}{n \cdot 4^n}$ are given by

(a) $R = 2,\ \ I = \left(-\dfrac{1}{2},\ \dfrac{7}{2}\right]$

(b) $R = 2,\ \ I = [-4,\ \ 4]$

(c) $R = 2,\ \ I = \left[-\dfrac{1}{2},\ \dfrac{7}{2}\right)$

(d) $R = 4,\ \ I = (-4,\ \ 4]$

(e) $R = 4,\ \ I = \left[-\dfrac{1}{2},\ \dfrac{7}{2}\right)$

13. If the region enclosed by the curves $y = e^x$, $x = 0$, $x = \ln 2$ and $y = 0$ is revolved about the line $y = -1$, then the volume of the solid generated is equal to

(a) $\frac{7\pi}{2}$

(b) $\frac{\pi}{2}$

(c) $\frac{9\pi}{2}$

(d) π

(e) $\frac{5\pi}{2}$

14. If $ax + bx^2 + cx^3$ is the sum of the first three terms of the Maclarium series of $e^{2x} \sin x$, then $a + b + c =$

(a) $\frac{29}{6}$

(b) $\frac{7}{3}$

(c) $\frac{5}{6}$

(d) $\frac{14}{3}$

(e) $\frac{31}{6}$

15. The sequence $\left\{(2n+1)\sin\frac{7}{n}\right\}$

(a) Converges to 14

(b) Converges to $\frac{7}{2}$

(c) Converges to 0

(d) Converges to $\frac{2}{7}$

(e) Diverges

16. If $x > e$, then $\frac{d}{dx}\int_1^{\sqrt{\ln x}} e^{t^2}\,dt =$

(a) $\frac{1}{2\sqrt{\ln x}}$

(b) $\frac{4}{\sqrt{\ln x}}$

(c) 0

(d) x

(e) $2x$

17. $\displaystyle\int \frac{x^2+x+3}{(x-1)\,(x^2+2x+2)}\;dx =$

(a) $\ln|x-1| - \tan^{-1}(x+1) + c$

(b) $x + \ln|x-1| + \tan^{-1}(x+1) + c$

(c) $\ln(x-1)^2 - \tan^{-1}(x+1) + c$

(d) $2x + \ln|x-1| - 3\tan^{-1}(x+1) + c$

(e) $\ln|x-1| + 2\tan^{-1}(x+1) + c$

18. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, n^5\, 6^n}{(n+1)!}$

(a) is absolutely convergent

(b) is conditionally convergent

(c) is divergent by the ratio test

(d) is divergent by the test for divergence

(e) is convergent by the integral test

19. The value of the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n}\left(\cos \frac{i\pi}{2n}\right)^2$ on the interval $\left[0, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{8}$

(b) $\frac{\pi}{2}$

(c) $1 + \frac{\pi}{8}$

(d) $1 + \frac{\pi}{2}$

(e) $-\frac{1}{4} + \frac{\pi}{8}$

20. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(a) Converges conditionally

(b) Converges absolutely

(c) Diverges

(d) Converges by the integral test

(e) Converges by the root test

21. $\int_0^{\frac{\pi}{4}} \sqrt{\dfrac{1+\sin x}{1-\sin x}}\ dx =$

(a) $\ln(2+\sqrt{2})$

(b) $\ln(2\sqrt{2}-1)$

(c) $\ln\sqrt{2}$

(d) $\ln(1+2\sqrt{2})$

(e) $\ln(1+\sqrt{2})$

22. The improper integral $\int_{-\infty}^{\infty} e^{-|x|}\ dx$

(a) Converges to 2

(b) Converges to $\dfrac{1}{2}$

(c) Converges to 1

(d) Converges to 0

(e) Diverges

23. Using the power series of $\int \frac{dx}{1+x^2}$, then the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$ is equal to

(a) $\frac{\pi}{4} - \frac{2}{3}$

(b) $\frac{\pi}{4} - \frac{1}{3}$

(c) $\frac{\pi}{4} + \frac{2}{3}$

(d) $\frac{\pi}{4} + \frac{1}{3}$

(e) $\frac{\pi}{4} + \frac{4}{3}$

24. The improper integral $\int_0^1 \frac{1}{e^x - 1}\, dx$

(a) Diverges

(b) Converges to $\ln(e-1)$

(c) Converges to $\ln(1-e^{-1})$

(d) Converges to 1

(e) Converges to 0

25. If $\int_0^{\pi} x\, f(\sin x)\, dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x)\, dx$, then $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx =$

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi^2}{16}$

(d) π

(e) π^2

26. For the convergent alternating series $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^3}$, what is the smallest number of terms needed to guarantee that S_n approximates S within $\frac{1}{125} \times 10^{-6}$?

(a) 499

(b) 599

(c) 488

(d) 198

(e) 408

27. An electric cable is hung between two towers that are 200 feet apart. If the cable takes the shape of a curve whose equation is

$$y = 50\cosh(x/50), \quad -100 \leq x \leq 100,$$

then the length of the cable between the two towers is equal to

(a) $50\left(e^2 - e^{-2}\right)$

(b) $100\left(e^2 + e^{-2}\right)$

(c) $50\left(e - e^{-1}\right)$

(d) $100\left(e + e^{-1}\right)$

(e) $50\left(e^2 - e^{-2}\right)^2$

28. Using the substitution $t = \tan\left(\frac{x}{2}\right), \quad 0 < x < \pi$, we get

$$\int \frac{2\,dx}{\sin x(1+\cos x)} = A\ln\left(\tan\frac{x}{2}\right) + B\tan^2\frac{x}{2} + c \text{ where } A + 2B =$$

(a) 2

(b) 1

(c) 3

(d) 0

(e) 4