King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

Math 102 – Calculus II

Exam – II
2007–2008 (073)

Tuesday, August 12, 2008 **Allowed Time: 2 Hours**

Name: _____

ID Number: _____

Section Number: _____ **Serial #:** _____

Instructions:

1. Write neatly and legibly. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification.
3. **Calculators and Mobiles are not allowed**.
4. Make sure that you have 9 different problems as shown in the table below (7 pages + cover page).

Page #	Grade	Maximum
Page 1 (1, 2, 3)		14
Page 2 (4, 5(a,b))		14
Page 3 (6(a,b))		14
Page 4 (6(c,d))		14
Page 5 (6(e,f))		14
Page 6 (7(a,b,c))		15
Page 7 (8,9(a,b))		15
Total:		**100**

1. [**6 points**] Find the average value of the function $f(x) = \sec x \tan^3 x$ on the interval $\left[0, \frac{\pi}{3}\right]$.

2. [**4 points**] Determine whether the series $\sum_{n=1}^{\infty} \frac{3n^2 + 7n + 4}{\sqrt{16n^4 + 3n + 2}}$ is convergent or divergent. Justify your answer.

3. [**4 points**] Write out the form of the partial fraction decomposition of $\frac{3x^2 + 9}{x^3(x+1)(x^2+16)^2}$. [Do not determine the numerical values of the coefficients].

4. [**8 points**] Using the method of cylindrical shells, **set up, but do not evaluate**, an integral for the volume of the solid obtained by rotating the region bounded by the curves $x = -y^2 + 2y$ and $x + y = 0$ about the line $y = -5$. [You must sketch the region and a typical rectangle].

5. [**6 points**] Let $\{S_n\}$ be the sequence of partial sums of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n + 2}$.

   (a) Find a formula for S_n.

   (b) Use part (a) to find the sum of the series.

6. Evaluate the integrals in (a) to (f):

(a) [**5 points**] $\displaystyle\int \frac{\cos^3 x}{\sqrt{\sin x}}\, dx.$

(b) [**9 points**] $\displaystyle\int \frac{1}{(x-2)^2\sqrt{4x-x^2}}\, dx.$

(c) [**6 points**] $\int \ln \sqrt{x+1}\, dx.$

(d) [**8 points**] $\int \dfrac{5x^2 - x + 26}{(x-1)(x^2+9)}\, dx.$

(e) [**6 points**] $\displaystyle\int \frac{dx}{x + x^{3/2}}\,.$

(f) [**8 points**] $\displaystyle\int e^{-x} \sin x \; dx\,.$

7. (a) [**4 points**] List the first five terms of the sequence $\left\{\dfrac{e^n}{n^2}\right\}$, then determine whether the sequence is convergent or divergent. Justify your answer.

(b) [**5 points**] Find a formula of the general term a_n of sequence $\left\{1, \dfrac{2}{2^2-1^2}, \dfrac{3}{3^2-2^2}, \dfrac{4}{4^2-3^2}, \dfrac{5}{5^2-4^2}, \ldots\right\}$, then determine whether the sequence is convergent or divergent. If it converges, find the limit.

(c) [**6 points**] Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^2 + 2^n\sqrt{n}}{2^n\, n^2}$ converges or diverges. Justify your answer.

8. [**7 points**] Determine whether the integral $\displaystyle\int_0^{\pi/2} \frac{\cos 2x}{1-\sin 2x}\,dx$ is convergent or divergent. Justify your answer.

9. [**8 points**] Given the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

   (a) Confirm that the integral test is applicable and use it to show that the series is convergent.

   (b) How many terms are required to ensure that the sum of the series is accurate to within $\dfrac{1}{24} \times 10^{-12}$? Justify your anwer.