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Math 102- Calculus II

A PREREQUISITE HANDOUT

In this handout we give a list of problems about some topics studied in previous courses (Math 001, Math 002, and Math 101) that will be used often in Math 102. The main objective is to help students recall and master these topics and hence be able to go smoothly through the course. We will concentrate on the following topics:

- Completing the Square
- Evaluating limits at $+\infty$

1. Completing the Square

To complete the square of the quadratic

$$ax^2 + bx + c,$$

we proceed as follows:

Step 1: Take a as a common factor from the first two terms

$$a\left(x^2 + \frac{b}{a}x \right) + c.$$

Step 2: Inside the parentheses, add and subtract $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$:

$$a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 \right) + c\,.$$

Step 3: Now the first three terms inside the parentheses form a perfect square:

$$a\left[\left(x + \frac{b}{2a} \right)^2 - \left(\frac{b}{2a}\right)^2 \right] + c.$$

Step 4: Simplify to get

$$a\left(x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a}.$$

Exercises 1

Complete the square of each of the following.

1. $x^2 + 3x + 1.$
2. $x^2 - 4x + 7.$
3. $2x^2 + 9x - 1.$
4. $5x^2 - 7x + 3.$
5. $-3x^2 + 2x - 8.$
6. $-7x^2 - 14x.$
7. $4x^2 + 4x + 1.$
8. $5 - 4x - x^2.$

9. $9x^2+6x-8.$
10. $x^4+4x^2-1.$
11. $2x^6-6x^3+5.$
12. $x-6\sqrt{x}+9.$
13. $x^{2}+x+1.$

2. Evaluating Limits at $+\infty$

Recall that in Math 101 we used L'Hospital's Rule when we get the indeterminate forms $\frac{\infty}{\infty},\frac{0}{0},\infty-\infty,0\cdot\infty,1^{\infty},0^{0},\infty^{0}.$

A Useful Fact: (a polynomial over a polynomial) $a_n \neq 0,\ b_m \neq 0.$

$$\lim_{x\to+\infty}\frac{a_n x^{n}+a_{n-1}x^{n-1}+\cdots+a_1 x+a_0}{b_m x^{m}+b_{m-1}x^{m-1}+\cdots+b_1 x+b_0}=$$

$$\begin{cases}\dfrac{a_n}{b_m} & \text{if } n=m\\ 0 & \text{if } n<m\\ \dfrac{a_n}{b_m}\times\infty & \text{if } n>m\end{cases}$$

Exercises 2

Evaluate the following limits.

1. $\lim_{x\to+\infty}\dfrac{x^2-2x+13}{4x^2+x}.$
2. $\lim_{x\to+\infty}\dfrac{x-1}{x^5+x-15}.$
3. $\lim_{x\to+\infty}\dfrac{x^3-3x^2-6x-6}{x^2+4x+3}.$
4. $\lim_{x\to+\infty}\dfrac{x^3-3x^2-6x-6}{-2x^2+4x+3}.$
5. $\lim_{x\to+\infty}\left(1+\dfrac{3}{x}\right)^{x}.$
6. $\lim_{x\to+\infty}\left(1-\dfrac{2}{5x}\right)^{x}.$
7. $\lim_{x\to+\infty}\left(1+\dfrac{\pi}{3x}\right)^{3x}.$
8. $\lim_{x\to0}(1+x)^{2/x}.$
9. $\lim_{x\to0}(1-3x)^{3/x}.$
10. $\lim_{x\to0}(1+ax)^{b/x}.$

11. $\lim_{x\to+\infty}\left(x-\sqrt{x^2-1}\right)$.
12. $\lim_{x\to+\infty}\left(\ln(2x)-\ln(x+2)\right)$.
13. $\lim_{x\to+\infty}\tan^{-1}\left(\frac{5x}{5x+10}\right)$.
14. $\lim_{x\to+\infty} x\sin\left(\frac{1}{x}\right)$.
15. $\lim_{x\to+\infty} x^2\sin\left(\frac{2}{x}\right)$.

Answers:

Exercises 1

1) $\left(x+\frac{3}{2}\right)^2-\frac{5}{4}$	**6)** $-7(x+1)^2+7$	**11)** $2\left(x^3-\frac{3}{2}\right)^2+\frac{1}{2}$
2) $(x-2)^2+3$	**7)** $(2x+1)^2$	**12)** $\left(\sqrt{x}-3\right)^2$
3) $2\left(x+\frac{9}{4}\right)^2-\frac{17}{8}$	**8)** $9-(x+2)^2$	**13)** $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$
4) $5\left(x-\frac{7}{10}\right)^2+\frac{11}{20}$	**9)** $(3x+1)^2-9$	
5) $-3\left(x-\frac{1}{3}\right)^2-\frac{23}{3}$	**10)** $\left(x^2+2\right)^2-5$	

Exercises 2

1) $\frac{1}{4}$	**6)** $e^{-2/5}$	**11)** 0
2) 0	**7)** e^{π}	**12)** $\ln 2$
3) $+\infty$	**8)** e^2	**13)** $\frac{\pi}{4}$
4) $-\infty$	**9)** e^{-9}	**14)** 1
5) e^3	**10)** e^{ab}	**15)** $+\infty$