

1. Evaluate the limit if it exists.

(a) $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$. (4 points)

(b) $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 1} - \sqrt{2}}{1 - x}$. (6 points)

(c) $\lim_{x \rightarrow 0^+} \frac{3}{x} \left(\frac{1}{4 + x} - \frac{1}{4 - x} \right)$. (6 points)

(d) $\lim_{x \rightarrow 2^-} ([x - 1] - x^2)$, where $[\cdot]$ denotes the greatest integer function. (3 points)

$$(e) \lim_{x \rightarrow +\infty} \frac{\cos^2 x}{x^3}.$$

(6 points)

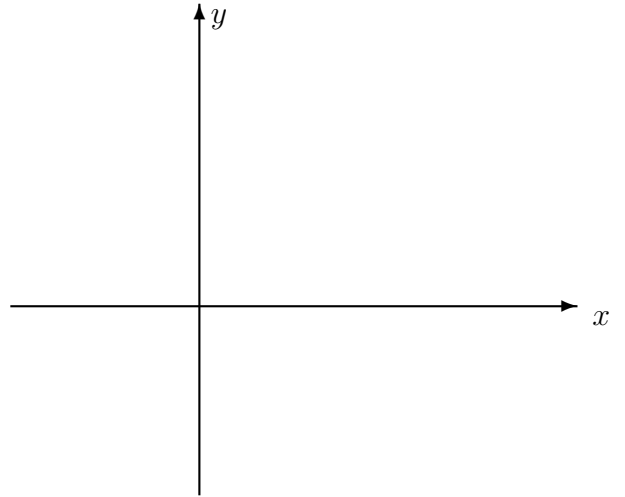
$$(f) \lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 2x}).$$

(8 points)

2. Use the graph of $f(x) = \frac{1}{x}$ to find a number δ such that

$$\left| \frac{1}{x} - \frac{1}{3} \right| < \frac{1}{5} \quad \text{whenever} \quad |x - 3| < \delta.$$

(7 points)



3. Consider the function

$$f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 1 \\ e - \ln x & \text{if } x > 1 \end{cases} .$$

- (a) Is f continuous from the left at 0. Justify.

(6 points)

- (b) Is f continuous at 1. Justify.

(6 points)

4. Where is the function $f(x) = \frac{\sin\left(\frac{1}{x}\right)}{e^x - 2}$ continuous. (6 points)

5. Show that the equation $e^x = -1 - 2x$ has a root in the interval $(-1, 0)$. (8 points)

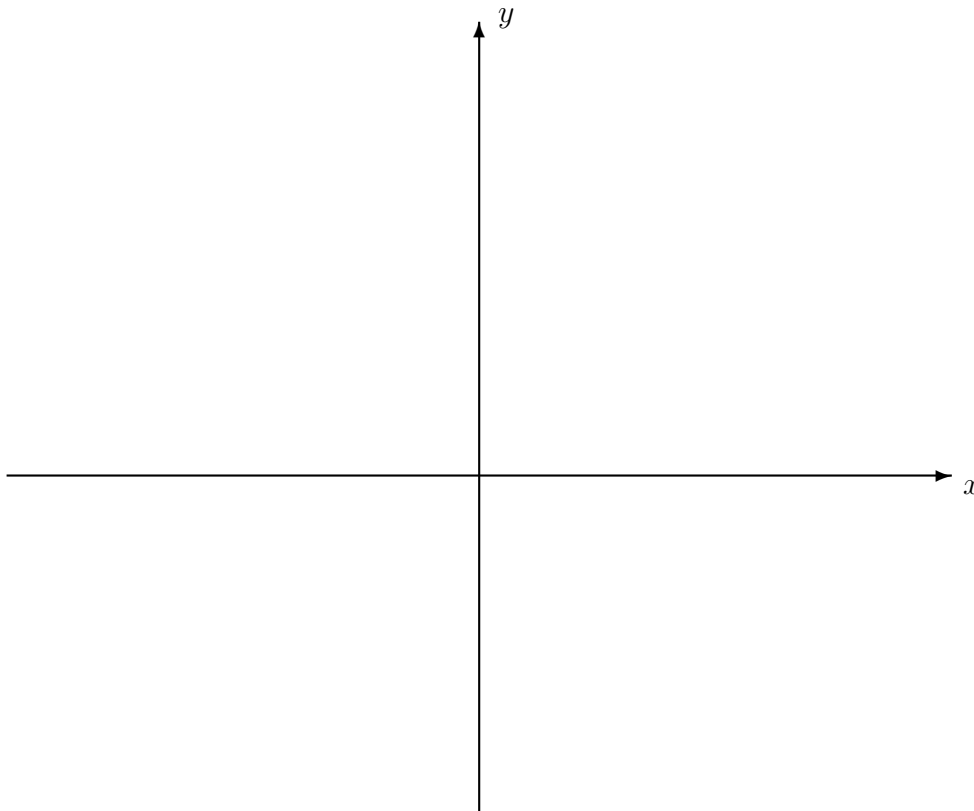
6. Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{4x - 1}{x^3 - 8x^2}. \text{ Explain.}$$

(9 points)

7. Sketch the graph of a function f that satisfies all of the given conditions:

$$f(0) = 0, f(2) = 1, \lim_{x \rightarrow 2^+} f(x) = 0, \lim_{x \rightarrow 2^-} f(x) = 2, \\ \lim_{x \rightarrow +\infty} f(x) = 4, \lim_{x \rightarrow -\infty} f(x) = -1, \lim_{x \rightarrow -2} f(x) = +\infty.$$



(8 points)

8. Find an equation of the tangent line to the curve $y = \frac{1}{x-2}$ at the point $\left(4, \frac{1}{2}\right)$.
[You must use limits] (8 points)

9. The position function of a particle moving in a straight line is given by the equation of motion $s = t^3 - 2t$, where t is measured in seconds and s in meters.

(a) Find the average velocity of the particle over the time interval $[1, 3]$. (3 points)

(b) Use limits to find the instantaneous velocity of the particle when $t = 2$. (6 points)