1. (a) [**3 points**] Write the following statement as a limit:

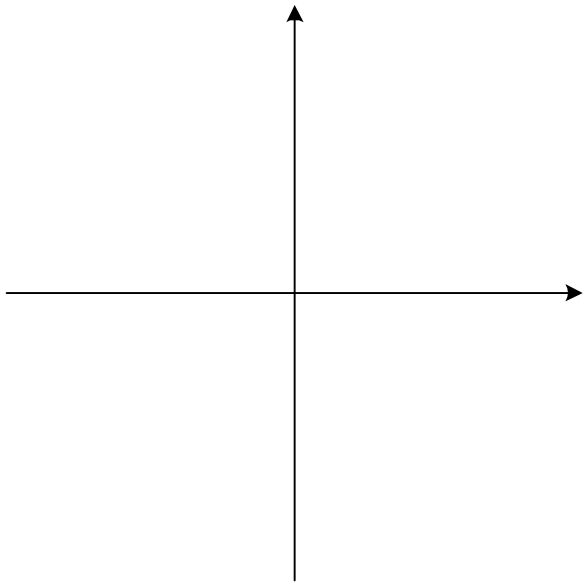
"$f(x)$ increases without bound as x approaches a from the left".

(b) [**4 points**] TRUE or FALSE: "If f has a domain $[0, +\infty)$ and has no horizontal asymptote, then $\lim_{x\to+\infty} f(x) = +\infty$ or $\lim_{x\to+\infty} f(x) = -\infty$".

[If TRUE, state the reason. If FALSE, illustrate graphically].

(c) [**7 points**] Sketch the graph of a function f that satisfies the following conditions:

i. $f(-1) = 3$
ii. $\lim_{x\to-1^-} f(x) = 4$
iii. $\lim_{x\to-1^+} f(x) = -\infty$
iv. $f(3)$ is undefined
v. $\lim_{x\to3} f(x) = 2$
vi. $\lim_{x\to+\infty} f(x) = +\infty$
vii. $\lim_{x\to-\infty} f(x) = 0$



2. Find the limit if it exists.

(a) [**6 points**] $\lim_{x\to -4} \dfrac{x^3 - 16x}{x+4}$

(b) [**6 points**] $\lim_{x\to 12} \dfrac{|12 - x|}{x - 12}$

(c) [**6 points**] $\lim_{x\to 3} g(x)$, where $2x - 1 \le g(x) \le x^2 - 5x + 11$

(d) [**6 points**] $\lim_{x\to 6^+} \tan^{-1}(\ln(x-6))$

3. [**8 points**] Using the ϵ, δ definition of limit, prove that $\lim_{x \to 1} \left(-1 + \frac{3}{2}\, x \right) = \frac{1}{2}$

4. [**8 points**] Let $f(x) = \begin{cases} \sqrt{x+2} & \text{if} \quad -2 \leq x \leq 2 \\ x^3 - 2x & \text{if} \quad x > 2\,. \end{cases}$ Is f continuous at $x = 2$. If not, what kind of discontinuity does f have at $x = 2$. Justify your answers.

5. [**6 points**] Where is the function $f(x) = \dfrac{1}{3 - \sqrt{x}}$ continuous?

6. [**8 points**] Show that the equation $e^{-x} = 2 - x$ has a root in the interval $(1, 2)$.

7. (a) [**8 points**] Find $\lim_{x \to +\infty} (\sqrt{x^2 + 1} - x)$.

(b) [**8 points**] Find the horizontal asymptotes of $f(x) = e^{x - x^2}$.

8. [**8 points**] Find an equation of the tangent line to the curve $y = \dfrac{1}{x^2 - x}$ at the point $\left(2, \dfrac{1}{2}\right)$. [You must use limits]

9. The displacement (in meters) of a particle moving in a straight line is given by the equation $s(t) = 3t^2 - 4t + 1$, where t is measured in seconds.

   (a) [**2 points**] Find the average velocity over the time interval $[0, 3]$.

   (b) [**6 points**] Use limits to find the instantaneous velocity when $t = 2$.