Dr. Ibrahim Al-Rasasi

MATH 101- REVIEW PROBLEMS
(31 Problems, 3 pages)

1. Find the following limits:
   a. $\lim_{x\to 3^+} \ln(x-3)$. **Answer:** $-\infty$
   b. $\lim_{x\to 7^-} \ln(7-x)$. **Answer:** $-\infty$
   c. $\lim_{x\to 0} \ln|x|$. **Answer:** $-\infty$
2. Sketch the graph of the following functions:
   a. $f(x)=\dfrac{x^2+3x+2}{x+2}$.
   b. $g(x)=\dfrac{|x-2|}{x-2}$.
   c. $h(x)=\begin{cases}\dfrac{1}{x} & \text{if } x\geq 2\\ x^2 & \text{if } x<2\end{cases}$
3. Find $\lim_{x\to 2}\dfrac{x^3-8}{x^4-16}$. **Answer:** 3/8.
4. Find the following limits:
   a. $\lim_{x\to 3^+}\dfrac{6x}{x-3}$.
   b. $\lim_{x\to 3^-}\dfrac{6x}{x-3}$.
   c. $\lim_{x\to 3}\dfrac{6x}{x-3}$.
5. Find the following limits:
   a. $\lim_{x\to 0^+}\dfrac{1}{1-e^x}$. **Answer:** $-\infty$
   b. $\lim_{x\to 0^-}\dfrac{1}{1-e^x}$. **Answer:** $+\infty$
   c. $\lim_{x\to 0}\dfrac{1}{1-e^x}$. **Answer:** DNE
6. Find the following limits:
   a. $\lim_{x\to -2/3}[\![x]\!]$. **Answer:** -1
   b. $\lim_{x\to 3}[\![x]\!]$. **Answer:** DNE
   c. $\lim_{x\to 1^-}[\![2x+1]\!]$. **Answer:** 2
7. Find the limit $\lim_{x\to 1}\dfrac{x-1}{\sqrt[3]{x}-1}$. **Answer:** 3
8. Using the ε,δ definition of limits, prove that $\lim_{x\to 12}(45-5x)=-15$.
   Answer: $\delta=\varepsilon/5$.
9. Where is each of the following functions continuous?
   a. $f(x)=\dfrac{x}{e^x-\ln x}$. **Answer:** $(0,+\infty)$.

b. $g(x)=\dfrac{x}{\sqrt{4-x^2}}$. **Answer:** $(-2,\ 2)$

10. Find all values of k that make the following functions continuous on $(-\infty,+\infty)$.

a. $f(x)=\begin{cases}x^3-2kx & \text{if } x\geq 1\\ kx+3 & \text{if } x<1\end{cases}$. **Answer:** $k=-2/3$.

b. $f(x)=\begin{cases}x^3-2kx & \text{if } x\neq 1\\ -k^2 & \text{if } x=1\end{cases}$. **Answer:** $k=1$.

11. Find the following limits:

a. $\lim_{x\to+\infty} 4\tan^{-1}(3-x^4)$. **Answer:** -2π.

b. $\lim_{w\to+\infty} \tan^{-1}\left(\dfrac{\sqrt{w}}{w+2}\right)$. **Answer:** 0.

12. Find the following limits:

a. $\lim_{t\to+\infty} e^{-t^2}$. **Answer:** 0

b. $\lim_{t\to+\infty} e^{t^3}$. **Answer:** $+\infty$

c. $\lim_{t\to+\infty} \dfrac{2}{\ln(t+3)}$. **Answer:** 0.

13. Find $\lim_{x\to+\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$. **Answer:** 1.

14. Find $\lim_{t\to+\infty} \dfrac{1}{t^3}\sin(3\sqrt{t})$. **Answer:** 0.

15. Find the vertical and horizontal asymptotes of the curve $y=\dfrac{x^2}{x^3+x}$. **Answer:** H. A.: $y=0$; No V. A.

16. Find the vertical and horizontal asymptotes of the curve $y=\dfrac{x^2+x-2}{x^2-4x+3}$.

Answer: H. A.: $y=1$; V. A.: $x=3$.

17. Where is $f(x)=[\![x]\!]$ differentiable.

18. Find $f'(x)$ if $f(x)=|4-2x|$.

19. Find the limit $\lim_{x\to 1} \dfrac{\sin(1-x)}{x^2-1}$. **Answer:** -1/2.

20. Find the limit $\lim_{x\to 0} \dfrac{\sin[\![x]\!]}{[\![x]\!]}$.

21. Find $\dfrac{dy}{dx}$:

a. $y=x^2\sqrt{1-x^2}$.

b. $y=\tan^3(x^5)$. **Answer:** $15x^4\tan^2(x^5)\sec^2(x^5)$.

c. $y=\cos(2\sin(\sqrt{x+3}))$.

22. Find the limit $\lim_{x\to 2} \dfrac{\cos(\pi\cos(\pi x))+1}{x-2}$. **Answer:** 0.

23. Find the slope of the curve $x^3 + 3xy^2 - y = 3$ at $(1,1)$. **Answer:** $-6/5$.
24. Find the equation of the tangent line to the curve $y = \tan^{-1}(x + \sqrt{1+x^2})$ at $x = 0$. **Answer:** $y = \frac{1}{2}x + \frac{\pi}{4}$.
25. Find $f^{(n)}(x)$ if $f(x) = xe^x$. **Answer:** $(x+n)e^x$.
26. Find $D^{100}\sinh(3x)$. **Answer:** $3^{100}\cosh(3x)$.
27. Find y' if $y = (\sin x)^{\cos x}$. **Answer:** $(\sin x)^{\cos x}[\cos x \cot x - (\sin x)\ln(\sin x)]$.
28. Find y' if $y = x^{(x^x)}$.
29. If $\cosh x = 3$, then find $\cosh(2x)$. **Answer:** 17.
30. Use linear approximation to estimate the value of $\ln(1.02)$.
31. Find all numbers c that satisfy the conclusion of Rolle's Theorem for $f(x) = e^{x^3 - x}$ over the interval $[0,1]$.