King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Calculus II
EXAM I
Semester II, Term 082
Tuesday, March 31, 2009
Net Time Allowed: 120 minutes

MASTER VERSION

1. Using four rectangles and right end points, the estimated area under the graph of $f(x) = 1 + \frac{x^2}{4}$ from $x = -2$ to $x = 6$ is

(a) 36

(b) 40

(c) 18

(d) 24

(e) 34

2. $\int \frac{\cos^2 t}{1 + \sin t}\, dt =$

(a) $t + \cos t + c$

(b) $1 + \cos t + c$

(c) $\frac{1}{2}t^2 - \cos t + c$

(d) $t - \sin t + c$

(e) $t - \frac{1}{2}\sin^2 t + c$

3. $\int (2-\sqrt{x})^2\,dx =$

(a) $4x - \frac{8}{3}\sqrt{x^3} + \frac{1}{2}x^2 + c$

(b) $\frac{(2-\sqrt{x})^3}{3} + c$

(c) $4x + \frac{1}{2}x^2 + c$

(d) $4x - x^{2/3} + \frac{1}{2}x^2 + c$

(e) $4x - 6x^{3/2} + x^2 + c$

4. $\int_4^{10} \frac{x}{x^2-4}\,dx =$

(a) $\frac{3}{2}\ln 2$

(b) $3\ln 2$

(c) $\frac{3}{4}\ln 2$

(d) $\frac{1}{2}\ln 2$

(e) $3\ln 4$

5. If $F(x) = \int_{\frac{1}{2}}^{x} f(t)\ dt$ and $f(t) = \int_{\frac{1}{2}}^{t^2} \frac{\sqrt{1+u^2}}{u}\ du$, then $F''(1) =$

(a) $2\sqrt{2}$

(b) $\sqrt{2}$

(c) $\frac{\sqrt{2}}{2}$

(d) $3\sqrt{2}$

(e) $\frac{\sqrt{2}}{6}$

6. If $y = \int_{\sqrt{x}}^{x^3} \sqrt{t} \sin t\ dt$, then $\frac{dy}{dx} =$

(a) $3\sqrt{x^7}\sin(x^3) - \frac{1}{2\sqrt[4]{x}}\sin\sqrt{x}$

(b) $\sqrt{x^3}\sin x^3 - \sqrt[4]{x}\sin\sqrt{x}$

(c) $\sqrt{x}\sin x^3 - \frac{1}{\sqrt[4]{x}}\sin\sqrt{x}$

(d) $\sqrt{x^3}\sin\sqrt{x} - x^2\sin x^3$

(e) $x^3\sin\sqrt{x} - \sqrt{x}\sin x^3$

7. Using the definition of the Area and definite integral, the value of the limit $\lim_{n\to\infty} \sum_{i=1}^{n} e^{4+\frac{3i}{n}} \cdot \frac{2}{n}$ is [Hint: Express the limit as a definite integral].

(a) $\frac{2}{3}[e^7 - e^4]$

(b) e^4

(c) $\frac{2}{3}e^{7/4}$

(d) does not exist

(e) $e^{-3} + e^{-4}$

8. If $f(x) = \begin{cases} x & 0 \le x \le 1 \\ \sqrt{1-(x-1)^2} & 1 \le x \le 2 \end{cases}$, and definite ingeral is interpreted as an area, then the value of the integral $\int_0^2 f(x)dx$ is

(a) $(2+\pi)/4$

(b) $(1+\pi)/4$

(c) $(3+\pi)/4$

(d) $(4+\pi)/4$

(e) $(5+\pi)/4$

9. $\int_{-1}^{2}(x-2|x|)dx =$

(a) $-\frac{7}{2}$

(b) $-\frac{9}{2}$

(c) $-\frac{5}{2}$

(d) $-\frac{3}{2}$

(e) $\frac{5}{2}$

10. $\int e^{(2x^5+\ln x^4)}\,dx =$

(a) $\frac{1}{10}e^{2x^5}+c$

(b) $e^{(\ln x^4+2x^5)}+c$

(c) $\frac{x^5}{5}+\frac{1}{3}x^6+c$

(d) $e^{(x^4+2x^5)}+c$

(e) $\frac{10}{3}e^{(x^4+2x^5)}+c$

11. $\int (\sec^2 x) \tan(\tan x)\, dx =$

(a) $\ln|\sec(\tan x)| + c$

(b) $\ln|\tan(\tan x)| + c$

(c) $-\sec(\tan x) + c$

(d) $\ln|\sin(\tan x)| + c$

(e) $\ln(\sec^2 x) + c$

12. If $\int_0^1 f(x)dx = \pi$, then $\int_0^{\pi/4} f(\sin 2x) \cos 2x\, dx$ is

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) π

(d) 2π

(e) 4π

13. $\int_1^2 \frac{x^2 - 2x - 3}{x^4 - 3x^3}\, dx =$

(a) $\frac{7}{8}$

(b) $-\frac{1}{8}$

(c) $\frac{9}{8}$

(d) $-\frac{3}{8}$

(e) $\frac{5}{8}$

14. The volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y^2 = x$ about the x-axis is equal to

(a) $3\pi/10$

(b) $37\pi/10$

(c) $\pi/10$

(d) $\pi/6$

(e) $5\pi/6$

15. The base of a solid is the region s bounded by the curves $y = \sqrt{x}, y = 0, x = 1$ and $x = 2$. If the cross-sections of the solid perpendicular to the x-axis are squares with one side lying along the base, then the volume of the solid is

(a) $\frac{3}{2}$

(b) 2

(c) 1

(d) $\frac{1}{2}$

(e) $\frac{2}{3}(2\sqrt{2} - 1)$

16. The area enclosed by the line $x + 2y = 1$ and the parabola $y^2 = 4 - x$ is given by the definite integral

(a) $\int_{-1}^{3} (3 + 2y - y^2) dy$

(b) $\int_{-1}^{3} \left[\frac{1}{2}(1 - x) - \sqrt{4 - x}\right] dx$

(c) $\int_{-1}^{3} (y^2 - 2y - 3) dy$

(d) $\int_{-3}^{4} \left[\frac{1}{2}(1 - x) - \sqrt{4 - x}\right] dx$

(e) $\int_{-3}^{1} (3 - 2y + y^2) dy$

17. The volume of the solid obtained by rotating the region bounded by the curves $y = \frac{1}{x}, y = 0, x = 1$ and $x = 3$ about the line $y = 1$ is

(a) $2\pi\left(\ln 3 - \frac{1}{3}\right)$

(b) $\pi\left(\ln 3 + \frac{1}{3}\right)$

(c) $2\pi\left(\ln 3 - \frac{2}{3}\right)$

(d) $\frac{2\pi}{3}$

(e) $\pi\left(2\ln 3 - \frac{1}{3}\right)$

18. The area of the region in the right half of the plane bounded by the curves $y = 2x - 1,\ y = x^2$, and $y = -x$ is equal to

(a) $\int_0^{1/3}(x^2 + x)dx + \int_{1/3}^{1}(x^2 - 2x + 1)dx$

(b) $\int_0^{1}(x^2 - x + 1)dx$

(c) $\int_0^{1/3}(x^2 - x)dx + \int_{1/3}^{1}(x^2 + 2x - 1)dx$

(d) $\int_0^{1}(x^2 + x - 1)dx$

(e) $\int_0^{1/3}(x - x^2)dx + \int_{1/3}^{1}(x^2 - 2x - 1)dx$

19. If we use the definition of $\ln x$ as a definite integral, then an approximation of $\ln 2$ using two rectangles and the sample points to be midpoints is equal to

(a) $\frac{24}{35}$

(b) $\frac{13}{15}$

(c) $\frac{12}{35}$

(d) $\frac{11}{15}$

(e) $\frac{29}{35}$

20. The limit $\lim_{n\to+\infty} \sum_{i=1}^{n} \frac{1}{i+n}$ can be interpreted as the

(a) area under the graph of the function $y = \frac{1}{x}$ on $[2, 4]$

(b) area under the graph of the function $y = \frac{1}{x}$ on $[0, 3]$

(c) area under the graph of the function $y = \ln x$ on $[2, 4]$

(d) area under the graph of the function $y = \ln x$ on $[1, 3]$

(e) area under the graph of the function $y = x$ on $[2, 4]$